Energy Correction Factor and Coefficient of Restitution

Continuing efforts to refine existing procedures for damage analysis have included developments related to the topics of (1) an energy correction factor to approximate the effects of tangential friction forces in oblique collisions, and (2) restitution effects. In the following, a brief summary is presented along with measures of the magnitudes of the cited effects.

Energy Correction Factor

Crush properties of vehicles are measured and fitted for crush directions that are perpendicular to the involved end or side of a vehicle. However, in an oblique collision, a component of the tangential friction force acts to increase the effective crush resistance in the direction of crushing and thereby, the absorbed energy (**Figure 49**). Therefore, an Energy Correction Factor (ECF) is needed for applications of crush coefficients to oblique collisions.

In the early development of CRASH (**Reference 49**) the need was recognized and a simplistic ECF was defined in the form of $(1 + \tan^2 \alpha)$, where α is the angle of crushing relative to a perpendicular to the involved end or side of the vehicle.

As application experience increased and evaluations were made of results at large angles, α, the ECF was limited to the angular range of ±45 degrees, so that the maximum value of the ECF was limited to 2.000 (e.g., **Reference 48**).

On the basis of a recognition of the limitations on energy absorption that are imposed by realistic levels of tangential friction, a revised form of the ECF was proposed in 1986 (**Reference 61**). Applications of the ECF by the author since that time have generally been restricted to the angular range of ± 45 degrees so that only a limited evaluation of the effects of the revised ECF was possible.

The topic has recently been revisited. A detailed review of the earlier analytical assumptions and the corresponding derivation of relationships has led to the proposed form of the modified ECF being further revised, on a purely analytical basis, to the following:

$$ECF = (1.0 + \mu_v \tan \alpha) \qquad (1)$$

It is proposed that the angular range of the ECF should be limited to ± 60 degrees, so that the maximum value of ECF is limited to approximately 1.95.

A comparison of the proposed ECF with the original form is presented in **Figure 50**.

The analytical derivation of Equation (1) follows:

In **Figure 51**, the force component that resists crushing may be defined as

$$F_\alpha = F_N (\cos \alpha + \mu_v \sin \alpha) \qquad (2)$$

Note in **Figure 51** that the force component, F_1, is necessary for force equilibrium. However, since F_1 is perpendicular to the direction of crushing, it does not affect the absorption of energy.

The displacement along the direction of crushing, δ_α, is related to that perpendicular to the end or side surface in the following manner:

$$\delta_\alpha = \frac{\delta_N}{\cos \alpha} \qquad (3)$$

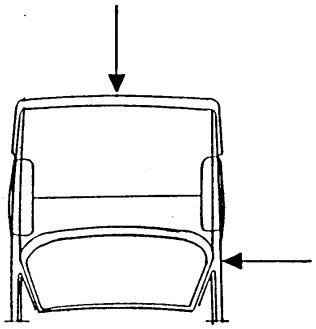
From (2) and (3), the work done against crush resistance may be expressed:

$$\int F_\alpha d\delta_\alpha = \frac{(\cos\alpha + \mu_v \sin\alpha)}{\cos\alpha} \int F_N d\delta_N$$
$$= (1 + \mu_v \tan\alpha) \int F_N d\delta_N \qquad (4)$$

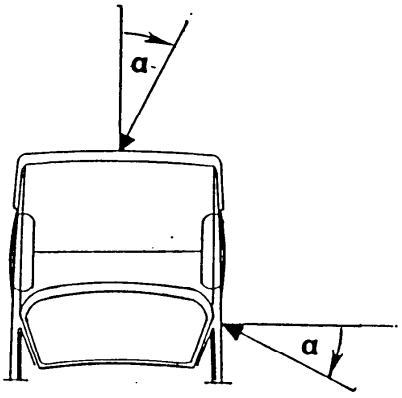
Thus, the indicated Energy Correction Factor,

$$ECF = (1 + \mu_v \tan \alpha) \qquad (5)$$

Clearly, the derived ECF must be evaluated by means of applications to oblique collisions over a range of angles, α. It is believed to be more realistic than the existing form (CRASH, EDCRASH) because of the fact that it limits the corresponding tangential force on the basis of an input value of tangential friction.



(a) Crush Properties are measured in directions that are perpendicular to the involved side or end.



(b) In applications, the directions of crushing are generally not perpendicular to the involved side or end.

Figure 49 Application of Fitted Crush Properties

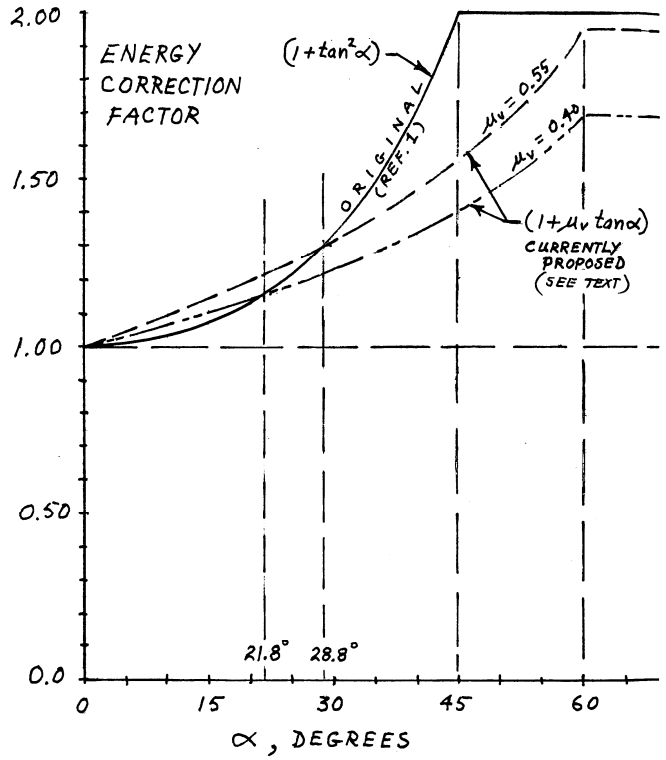


Figure 50 A Comparison of Proposed Forms of the Energy Correction Factor

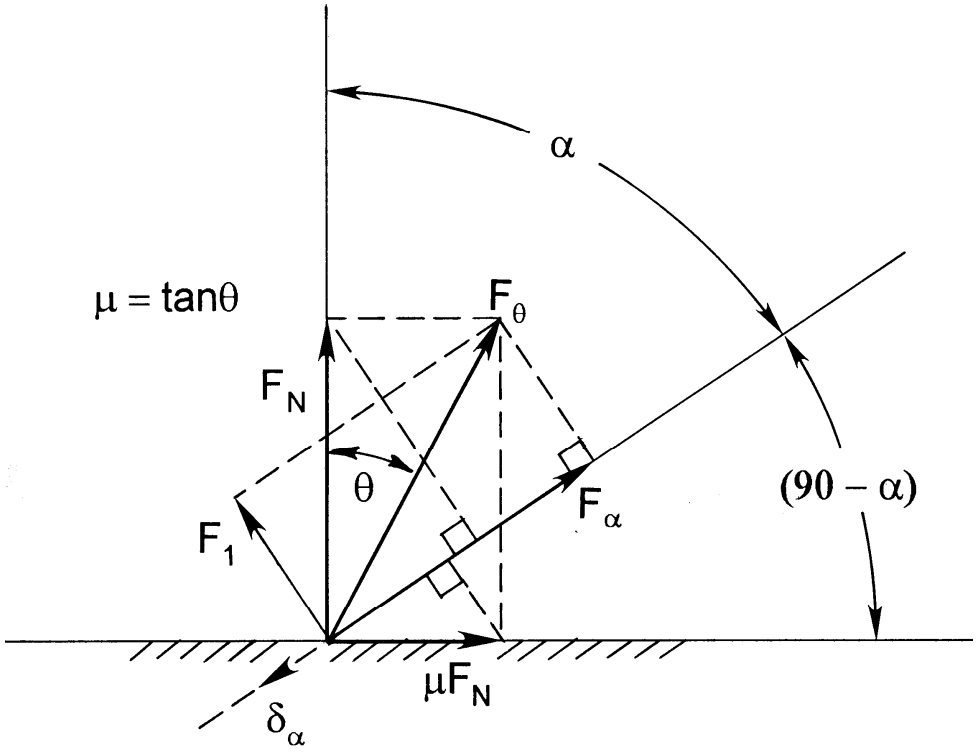


Figure 51 Force Components

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