Occupant Trajectory

Distance Traveled by an Ejected Vehicle Occupant or Component

A number of analytical relationships have been applied in the past for the purpose of interpreting the travel distances of ejected vehicle occupants or components. In the following, the related assumptions and the derivations of several such relationships are outlined. A summary comparison of the analytical relationships is presented in

Table 13. The analytical basis for a general-case analytical approach with an iterative solution procedure, which has been developed by McHenry Software, Inc., as the LAUNCH routine, is then briefly outlined.

Simple Ballistic Trajectory

An oversimplified analytical relationship that has sometimes been used to approximate the minimum speed of an ejected occupant or component is based on a ballistic trajectory with the following inherent assumptions:

1. The optimum launch angle (i.e., 45°) to determine the minimum speed for a given travel distance,
2. A landing at the same elevation as the launch, and
3. No movement on the ground after the landing.

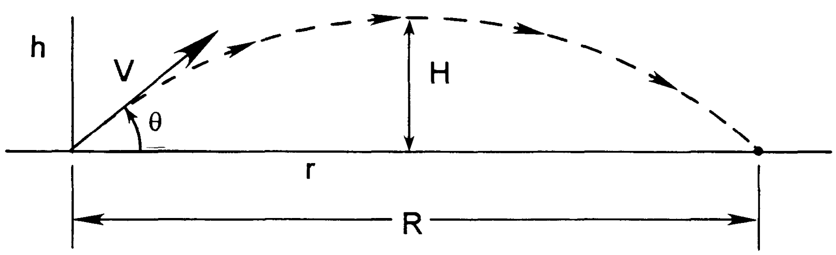

Figure 67 Simple Ballistic Trajectory

The assumed behavior is depicted in **Figure 67** and the cited relationship is derived as follows:

$$h = (V \sin\theta)\, t - \frac{1}{2} g t^2 \qquad (1)$$

$$h = 0 \text{ when } t = 0, \text{ and when } t = \frac{2V\sin\theta}{g}.$$

$$r = (V\cos\theta)\, t \qquad (2)$$

$$R = \frac{2V^2}{g}\cos\theta\sin\theta \qquad (3)$$

$$\frac{dR}{d\theta} = \frac{2V^2}{g}\left(\cos^2\theta - \sin^2\theta\right) \qquad (4)$$

$$\text{For } \frac{dR}{d\theta} = 0,\ \theta = 45°$$

$$R = \frac{V^2}{g} \qquad (5)$$

$$V_{min} = \sqrt{Rg} \quad \text{FT/SEC} \qquad (6)$$

$$= 5.675\sqrt{R} \quad \text{FT/SEC}$$

$$V_{min} = 3.869\sqrt{R}\ \text{MPH} \qquad (7)$$

$$\text{where R = Feet.}$$

The unrealistic nature of equation (7) becomes more apparent when the corresponding maximum elevation is determined.

From Equation (1),

$$\frac{dh}{dt} = V\sin\theta - gt \qquad (8)$$

$$\text{For } \frac{dh}{dt} = 0,\ t = \frac{V\sin\theta}{g}$$

Evaluation of equation (1) at

$$t = \frac{V\sin\theta}{g}\ \textit{yields}:$$

$$H = \frac{V^2\sin^2\theta}{2g} \qquad (9)$$

From equations (3) and (9), for $\theta = 45°$:

$$H = 0.25R \qquad (10)$$

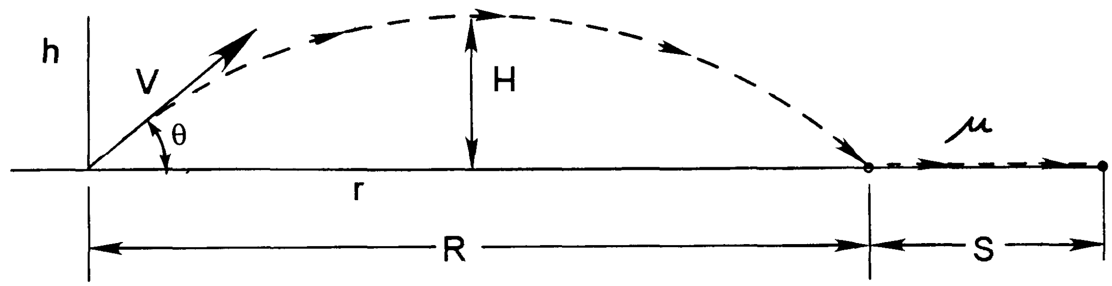

Figure 68 Ballistic Trajectory Followed by Sliding

Ballistic Trajectory Followed by Sliding

If it is assumed that the motion continues after landing (as depicted in **Figure 68**), with an average drag force equal to μ Mg, the following relationships can be developed:

$$R + S = \frac{2V^2}{g}(\sin\theta\cos\theta) + \frac{V^2\cos^2\theta}{2\mu g} \tag{11}$$

$$\frac{d(R+S)}{d\theta} = \frac{2V^2}{g}\left(\cos^2\theta - \sin^2\theta\right) - \frac{V^2}{\mu g}(\cos\theta\sin\theta) \tag{12}$$

The range $(R + S)$ has a maximum value when $\frac{d(R+S)}{d\theta} = 0$,

which occurs for the following value of θ :

$$\cos^2\theta - \sin^2\theta - \frac{\cos\theta\sin\theta}{2\mu} = 0 \tag{13}$$

$$\cos 2\theta - \frac{\sin 2\theta}{4\mu} = 0 \tag{14}$$

$$\theta = \frac{1}{2}\arctan(4\mu) \tag{15}$$

Using the following trigonometric relationships and Equation (15), Equation (11) can be solved for the required launch speed:

$$\cos^2\theta = \frac{(1+\cos 2\theta)}{2} \tag{16}$$

$$\sin\theta\cos\theta = \frac{\sin 2\theta}{2} \tag{17}$$

$$\tan\theta = \frac{\sin 2\theta}{(1+\cos 2\theta)} \tag{18}$$

$$V = \sqrt{g(R+S)\tan\theta} \quad \text{FT/SEC} \tag{19}$$

$$V = 3.869\sqrt{(R+S)\tan\theta} \quad \text{MPH} \tag{20}$$

Where R+S = Feet

Note that if $\mu \to \infty$, $\theta = 45°$, $S = 0$, and the results will be identical with Case 1 (simple Ballistic Trajectory).

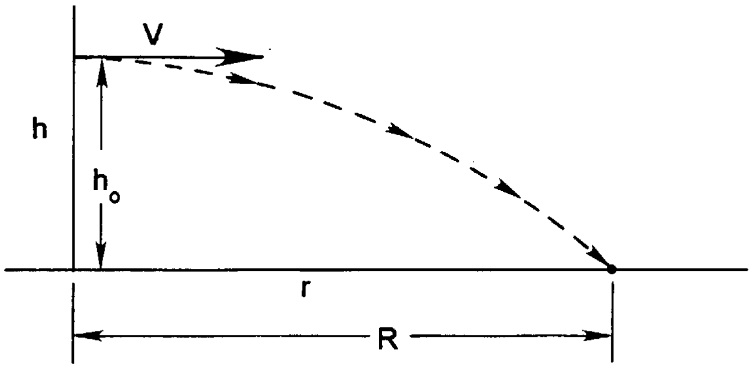

Figure 69 Simple Horizontal Launch

Simple Horizontal Launch

An oversimplified analytical relationship that has sometimes been used to approximate the maximum speed of an ejected occupant or component is based on a horizontal launch with the assumption of no movement on the ground after landing. The assumed behavior is depicted in **Figure 69.**

$$h = h_o - \frac{1}{2} g t^2 \quad (21)$$

$$\text{For } h = 0, \ t = \sqrt{\frac{2h_0}{g}} \quad (22)$$

$$R = Vt = V\sqrt{\frac{2h_0}{g}} \quad (23)$$

Solving for V,

$$V = R\sqrt{\frac{g}{2h_0}} \ \text{FT/SEC} \quad (24)$$

$$V = \frac{2.736R}{\sqrt{h_o}} \ \text{MPH} \quad (25)$$

where R, h_o = Feet

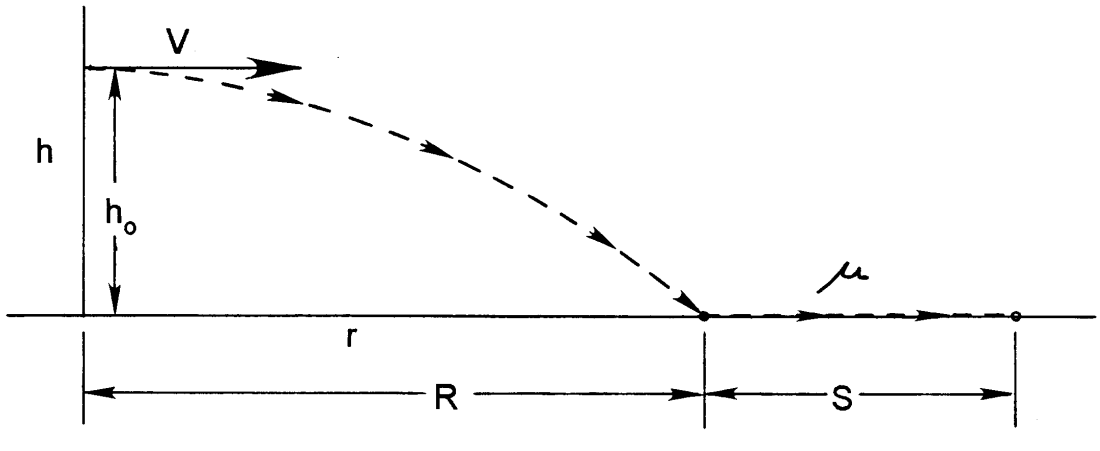

Figure 70 Horizontal Launch Followed by Sliding

Horizontal Launch Followed by Sliding

If it is assumed that the motion continues after landing (as depicted in **Figure 70**), with an average drag force equal to μ Mg, the following relationships can be developed:

$$R + S = V\sqrt{\frac{2h_0}{g}} + \frac{V^2}{2\mu g} \qquad (26)$$

Solution of equation (26) for V yields :

$$V = -\mu\sqrt{2h_0 g} + \sqrt{2\mu g\left(\mu h_0 + R + S\right)} \quad \text{FT/SEC} \qquad (27)$$

$$V = 5.4716 \quad \left\{-\mu\sqrt{h_0} + \sqrt{\mu\left(\mu h_0 + R + S\right)}\right\} \text{ MPH} \qquad (28)$$

where R, S, h_o = Feet

	1	2	3	4	
Quantity	Simple Ballistic Trajectory	Ballistic Trajectory Followed by Sliding	Simple Horizontal Launch	Horizontal Launch Followed by Sliding	UNITS
Total Horizontal Travel	R	R+S	R	R+S	FEET
Terrain Surface Deceleration	∞	μ	∞	μ	G Units
Elevation of Launch	0	0	h_0	h_0	Feet
Angle of Launch, θ	45°	½ arctan (4μ)	0.0°	0.0°	Degrees
Velocity of Launch	$3.869\sqrt{R}$	$3.869\sqrt{(R+S)\tan\theta}$	$\frac{2.736R}{\sqrt{h_0}}$	$5.4716\left\{-\mu\sqrt{h_o}+\sqrt{\mu(\mu h_0+R+S)}\right\}$	MPH
Horizontal Distance in Air	R	R	R	R	Feet
Horizontal Distance on Ground	0	S	0	S	Feet
Maximum Height Of Trajectory	0.25R	$\left(\frac{R+S}{2}\right)\tan\theta\sin^2\theta$	h_0	h_0	Feet
Distance to Max. Height	0.50R	0.50R	0	0	Feet
Time to Max. Height	$0.7071\sqrt{\frac{R}{g}}$	$\sin\theta\sqrt{\left(\frac{R+S}{g}\right)\tan\theta}$	0	0	Seconds
Time in Air	$1.4142\sqrt{\frac{R}{g}}$	$2\sin\theta\sqrt{\left(\frac{R+S}{g}\right)\tan\theta}$	$\sqrt{\frac{2h_0}{g}}$	$\sqrt{\frac{2h_0}{g}}$	Seconds
Total Time in Motion	$1.4142\sqrt{\frac{R}{g}}$	$\left(2\sin\theta+\frac{1}{\mu}\right)\sqrt{\left(\frac{R+S}{g}\right)\tan\theta}$	$\sqrt{\frac{2h_0}{g}}$	$\sqrt{\frac{2(\mu h_0+R+S)}{\mu g}}$	Seconds

Table 13 Summary of Analytical Relationships for Occupant Trajectory

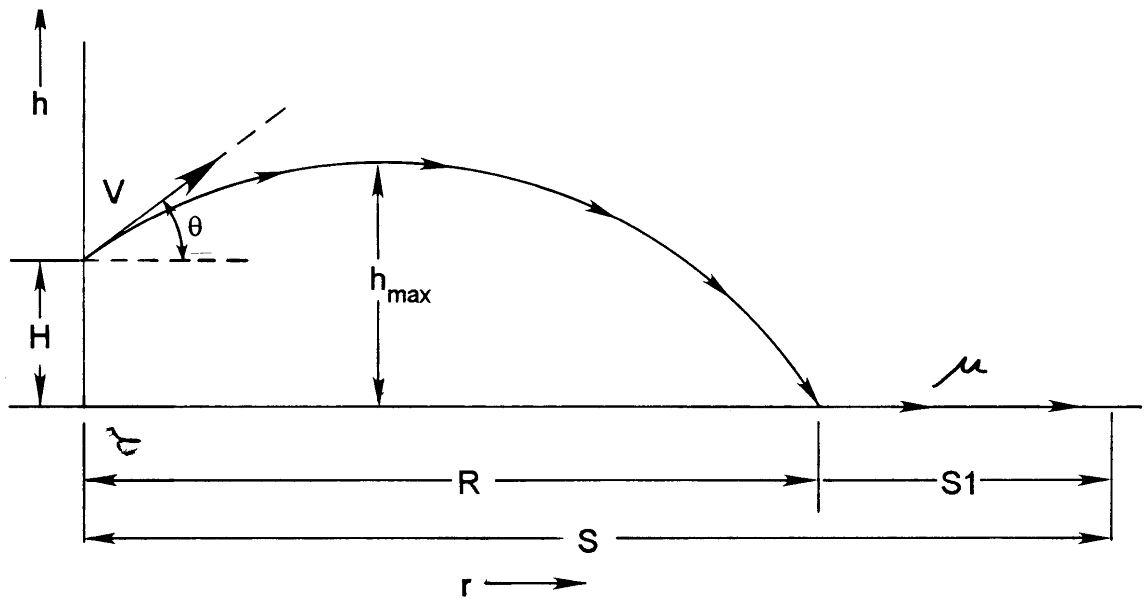

Figure 71 General Case Launch Analysis

LAUNCH Routine.

The LAUNCH routine, developed by McHenry Software, Inc., constitutes a general-case analytical approach, as depicted in **Figure 71.** An iterative solution procedure is used. The output format which is depicted in **Figure 72** ***and*** **Figure 73**, includes the option of either (a) specifying a launch angle or (b) permitting the LAUNCH routine to find the optimum angle (i.e., to determine the minimum speed for a given travel distance. The results of sample applications are also depicted in **Figure 74.**

LAUNCH Program

INPUTS:		
	65.00	Total Horizontal Travel, Ft.
	0.70	Terrain Surface Decel.,G-units
	3.50	Elevation of Launch, Ft.
Calculated Outputs:		
Theta(N)	32.21	Degrees, Angle of Launch
V(N)	25.22	MPH, Velocity of Launch
R(N)	43.26	Feet, Horizontal Distance in Air
S1(N)	21.74	Feet, Horizontal Distance on Ground
HMAX	9.54	Feet, Maximum Height of trajectory
R1	19.17	Feet, Distance to HMAX
T1	0.61	SEC, Time to Maximum Height
T2	1.38	SEC, Total time in air
T3	2.77	Sec, Total time in motion
N	12	Number of iterations

Figure 72 Sample Launch Output Format for optimum launch angle

LAUNCH Program

INPUTS:

	65.00	Total Horizontal Travel, Ft.
	0.70	Terrain Surface Decel.,G-units
	3.50	Elevation of Launch, Ft.
	15.00	Degrees, Angle of Launch

Calculated Outputs:

V(N)	26.72	MPH, Velocity of Launch
R(N)	33.22	Feet, Horizontal Distance in Air
S1(N)	31.78	Feet, Horizontal Distance on Ground
HMAX	5.10	Feet, Maximum Height of trajectory
R1	11.92	Feet, Distance to HMAX
T1	0.31	SEC, Time to Maximum Height
T2	0.88	SEC, Total time in air
T3	2.56	Sec, Total time in motion
N	11	Number of iterations

Figure 73 Sample Launch Output Format for 15 ° launch angle

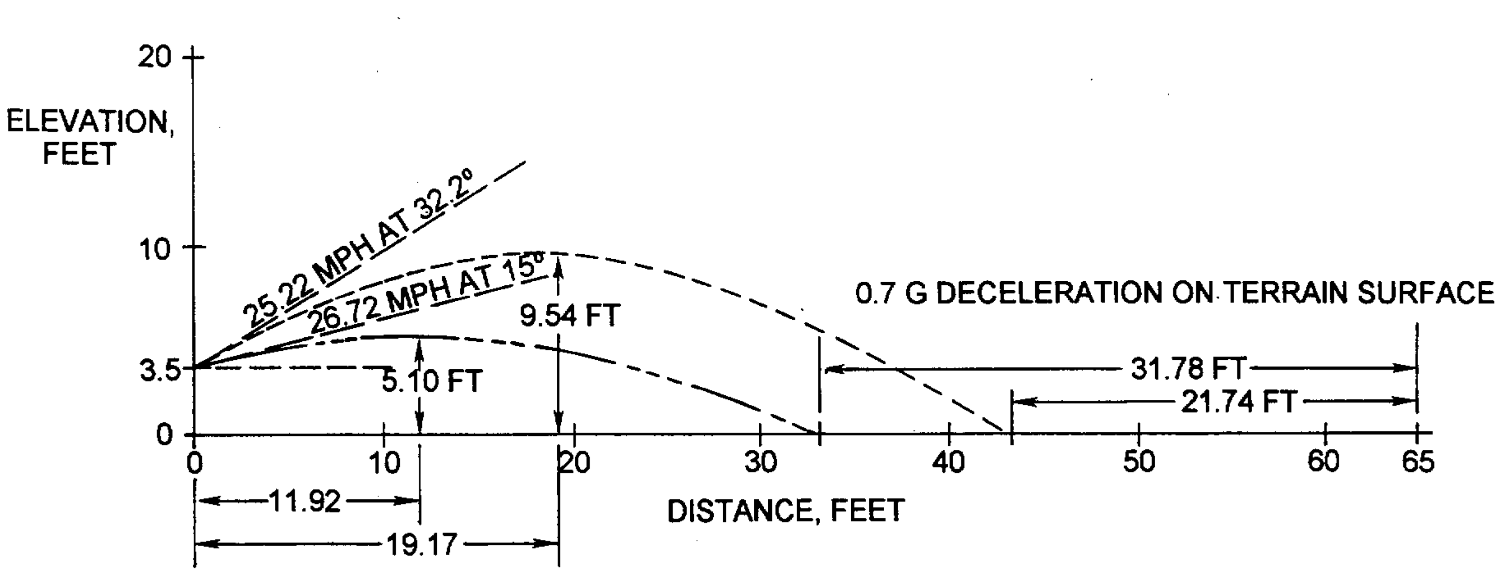

Figure 74 Results of Sample Applications of Launch