

McHenry

Accident Reconstruction

2008

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Trajectory Analysis

Introduction

When vehicles separate after a collision, they move to rest positions against resistance forces produced primarily by tire-ground friction. Secondary contacts may also occur with roadside obstacles and/or terrain features which sometimes can play significant roles in the dissipation of kinetic energy and can also produce redirections of the spinout trajectories. The task of analyzing the total energy dissipated as the vehicles travel from separation to their positions of rest is essential in preparing a comprehensive reconstruction of the collision.

On the basis of Newton's 2nd and 3rd laws, the total momentum of an isolated system of masses remains constant. This principal, which is referred to as the *Conservation of Momentum*, serves as the theoretical basis for reconstruction of impact speeds in vehicle-to-vehicle collisions.

For the moment we will assume that the system is isolated and will ignore the external forces produced by the tires and other possible sources, such as gouging and scraping of vehicle components on the ground, during the collision. The magnitudes of these external forces are normally small when compared with the magnitude of the forces of the collision. However, they should not be totally ignored.

A trajectory analysis is used to determine each vehicle's velocity and direction subsequent to the collision, thereby providing a definition of the **system** momentum at separation. This can then be used to define the **system** momentum at the instant of collision and thereby provide a procedure to determine the vehicle's impact speeds. This procedure for estimating impact velocities also directly provides estimates of the impact speed-changes (ΔV) in the form of the vector differences between impact and separation velocities for each vehicle.

Our presentation of trajectory analysis will begin with the simplest form of vehicle motion, linear motion without yawing:

Linear Motions Without Yawing

For the simplest case of straight-line travel without yawing rotation and with constant drag forces, the corresponding change in velocity can be approximated with analytical relationships for constant deceleration:

$$V = V_0 - at \quad (1)$$

From integration of (1),

$$S = V_0 t - 1/2 at^2 \quad (2)$$

$$t = \frac{(V_0 - V)}{a} \quad (3)$$

Substitution of (3) into (2) yields

$$S = \frac{(V_0^2 - V^2)}{2a} \quad (4)$$

Solution of (4) for V yields

$$V^2 = V_0^2 - 2aS \quad (5)$$

where

V	=	Velocity
V ₀	=	Initial velocity
a	=	Acceleration
t	=	Time
S	=	Distance

In applications of equation (5), a distinction must be made between the prevailing average friction coefficient, μ , and the deceleration, a . If the full 100% friction coefficient is utilized, i.e., wheels locked or pure lateral travel, $a = \mu g \text{ ft/sec}^2$.

For longitudinal motions with one or more wheels not fully locked and/or sideslip angles less than 90 degrees, the friction utilization will be less than 100 percent and, thereby, $a < \mu g \text{ ft/sec}^2$.

In the case of a vehicle coming completely to rest without further obstacle contacts, $V_f = 0$, and equation (5) becomes:

$$V_0^2 = 2aS \quad (6)$$

The decrease of speed with travel distance, for the case of constant deceleration, is depicted in **Figure 18**.

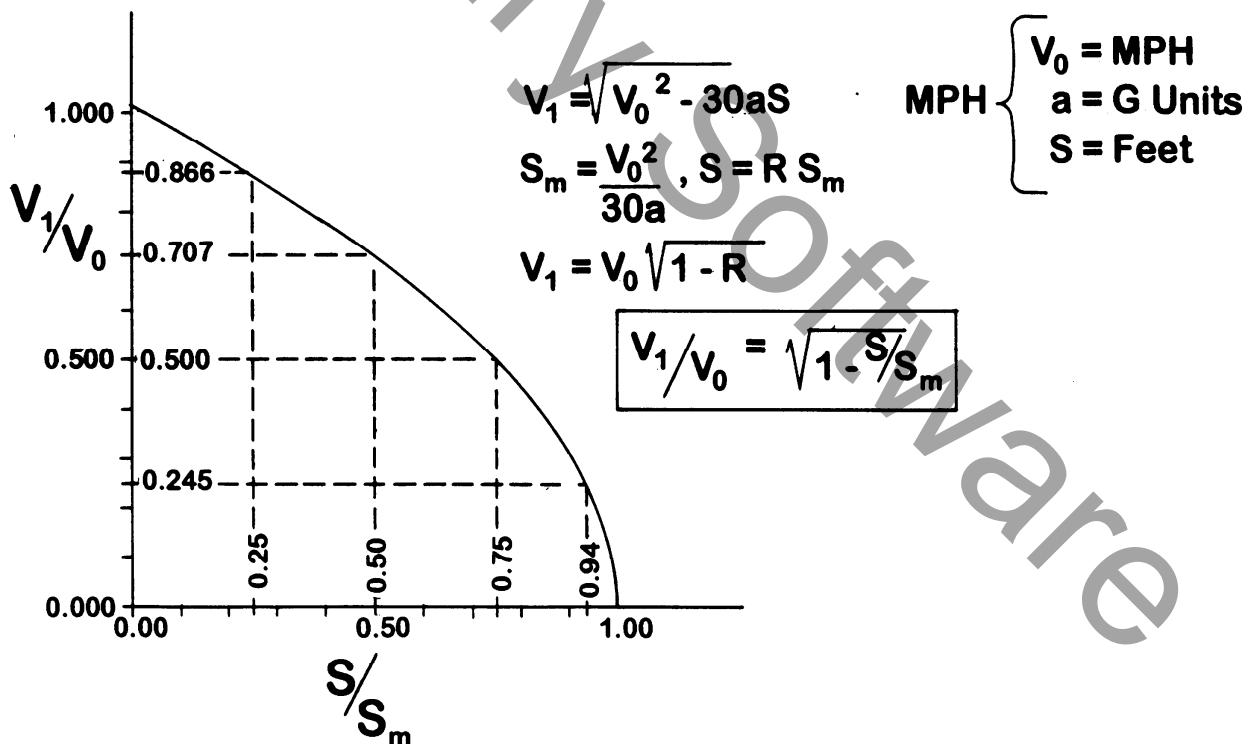
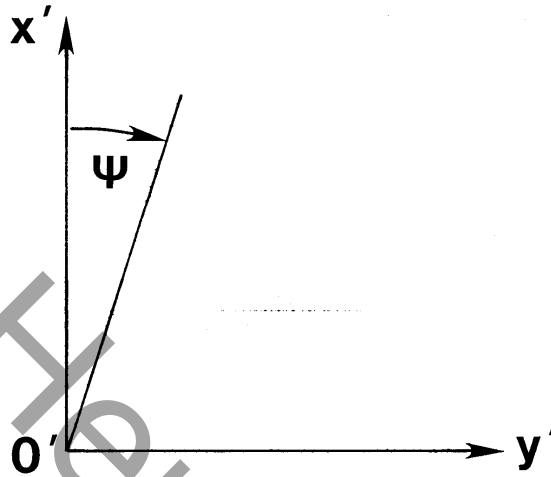


Figure 18 Speed Decrease with travel at Constant Deceleration

Spinouts on Flat, Uniform Surfaces

A space-fixed coordinate system is necessary to define measured spinout trajectories for analysis (e.g., **Figure 19**). The relationship of the X' and Y' axes in **Figure 19** reflects the aeronautical convention for three-dimensional coordinates, in which the Z' axis points downward. In the selected coordinate system, the Y' axis is directed to the right of the X' axis and the angle Ψ is measured in the clockwise direction with respect to the X' axis.



SPACE-FIXED COORDINATE SYSTEM

Figure 19 Space Fixed Coordinate system

While the position and orientation of the space-fixed reference coordinate system for a given case are arbitrary, it is generally desirable to relate them to permanent reference points at the accident scene (e.g., curb lines, utility poles, etc.).

The position data used in trajectory calculations refers to the location of the vehicle center of gravity, something which is not readily measurable at the accident scene. The reconstructionist must interpret tiremark evidence and/or measurements to wheel locations and from these data ascertain the center of gravity location.

In simple spinout motions, the actual distances traveled to rest can be approximated, with reasonable accuracy, by straight lines between the separation and rest positions. The separation velocities can be estimated on the basis of the total work done by each vehicle against tire-terrain friction forces between separation and rest. The rate of energy dissipation by tire forces is dependent on the heading direction of the vehicle in relation to its direction of motion and on the extent of rotational resistance at the individual tires. For example, in a broadside slide, all tires produce important resistance forces. In forward or backward motion, only those tires with applied brakes, damage effects, large steered angles or driveline braking produce significant drag forces. At a given sideslip angle, or over a limited range of a changing sideslip angle, the motion resistance can be approximated in the manner depicted in **Figure 20**. For the case of rotation about a vertical axis (i.e., yawing rotation) a vehicle alternates between the two conditions of motion.

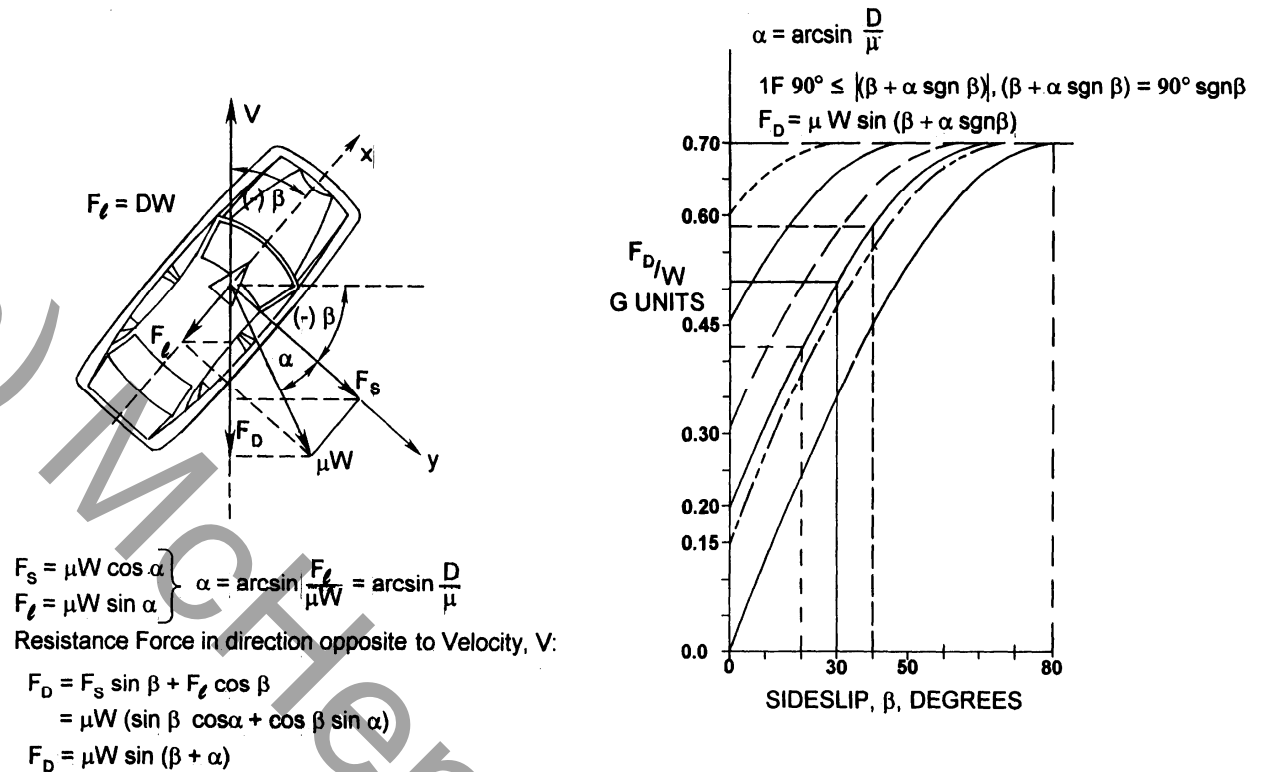


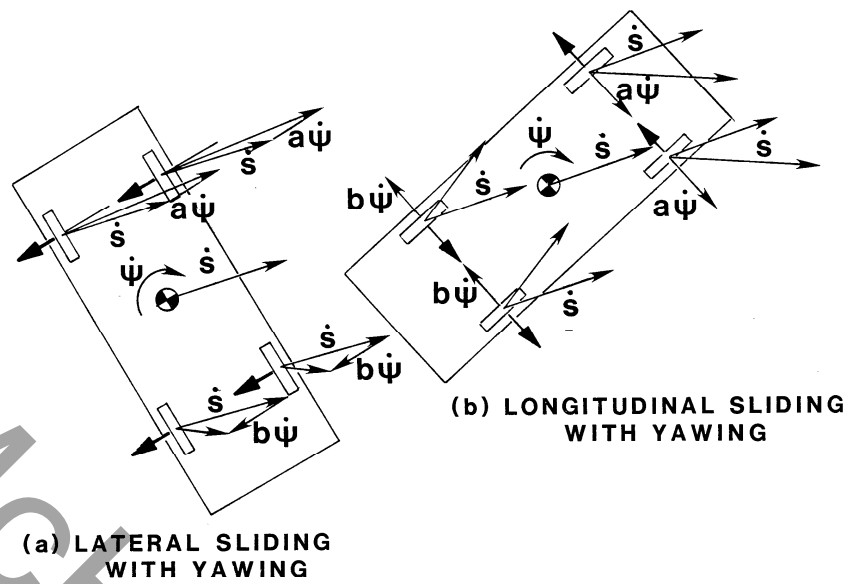
Figure 20 Approximation of Motion Resistance in Sideslip

Several different conditions of the vehicle tires may exist which can produce different extents of rotation during a spinout and therefore produce differing amounts of energy dissipation. Either the wheels may be freely rotating or there may be some resistance to rotation due to either vehicle damage or connection to a driveline (which produces the drag on the vehicle that is called ‘driveline braking’).

With freely rotating wheels, the linear and angular velocities of a vehicle are decelerated alternately as the heading direction changes with respect to the direction of motion. When the vehicle slides laterally, the side forces at the front and rear tires tend to have the same direction despite the existence of a yawing velocity (**Figure 21(a)**). Therefore, during this phase of motion, the angular velocity tends to remain constant while the linear velocity is decelerated. When the longitudinal axis is aligned with the direction of the linear velocity, the side forces at the front and rear tires act in opposite directions and the angular velocity is decelerated while the linear velocity tends to remain constant (**Figure 21(b)**). The general nature of the alternating decelerations is depicted in **Figure 22**.

The other situation which can occur is if one or more of the wheels has rotational resistance. In that situation the linear velocity is decelerated, generally at different rates, during both phases of the spinout motion. The amount and location of the wheel drag on a vehicle directly affects its behavior. A vehicle with all wheels locked tends to decelerate at a faster rate than one with less than all wheels locked. Also, the location of the locked wheel with respect to the velocity direction and heading (i.e., lateral vs. longitudinal) of a vehicle in a spinout affects the characteristics of it’s angular deceleration time history.

Differences which are related to wheel rotation drag become most apparent when either the amount of yaw rotation is greater than 60 degrees and/or the vehicle spends a significant amount of time in a near longitudinal side-slip. A random sampling of angular velocity time histories for four different cases with either none, one, two, three, or four wheels with drag is illustrated in **Figure 23** from **Reference 23**.



TIRE FORCES DURING SPINOUT WITH YAWING VELOCITY

Figure 21 Tire Forces During Spinout

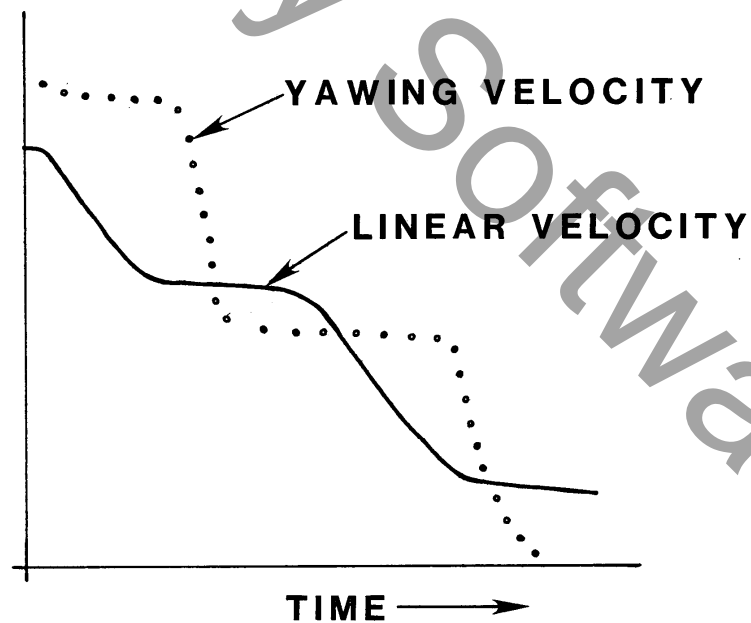


Figure 22 Linear and Angular (Yawing) Velocity vs. Time (no braking)

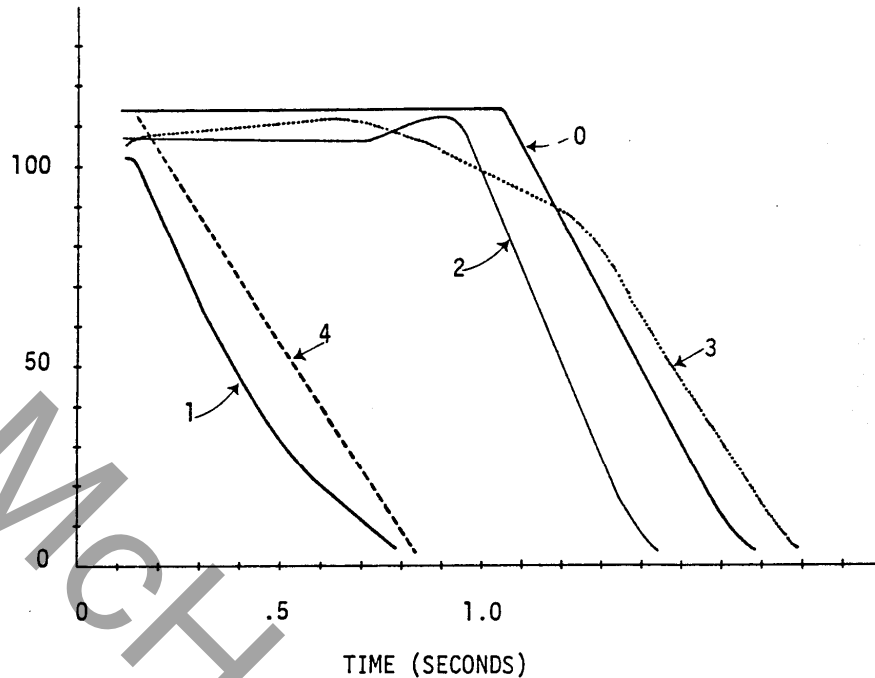


Figure 23 Random sampling illustrating angular velocity deceleration variations due drag
(For conditions with 0,1,2,3 and 4 wheels with drag (from Reference 23))

In the following an analytical procedure is presented for approximating the linear and angular velocities of a vehicle at the start of its motions subsequent to a collision, which was first defined by Marquard in **Reference 24**, and which served as a starting point for corresponding aspects of the CRASH computer program (**Reference 25**). The cited procedure takes into account the fact that the linear and angular (i.e., yaw rotation) displacements of a four-wheeled vehicle subsequent to a collision occur under conditions of intermittent deceleration when the wheels are free to rotate. By approximating the linear and angular deceleration rates of a vehicle with either (1) all wheels freely rotating or (2) all wheels locked during the different phases of a spinout motion, Marquard developed approximate relationships on the basis of the total linear and angular displacements during the travel from separation to rest. He then estimated the corresponding linear and angular velocities of the vehicle at separation from its collision partner, for two cited cases of rotational resistance at the wheels. The procedure has been generalized in the following to include intermediate conditions of rotational resistance at the wheels.

Combined Rotation and Translation Subsequent to Collision

A technique for generating initial estimates of the collision separation conditions:

INPUTS

X'_{CR}, Y'_{CR}, ψ_R = Rest position and orientation

(feet and degrees)

X'_{CS}, Y'_{CS}, ψ_S = Position and orientation at separation

(feet and degrees)

$a + b$ = Wheelbase, inches

k^2 = Radius of gyration squared for complete vehicle
in yaw, in²

μ = Nominal tire-ground friction coefficient

θ = Decimal portion of full deceleration

$$0 \leq \theta \leq 1.000$$

g = Acceleration of gravity

$$= 386.4 \text{ inches/sec}^2$$

$$1.0 \quad S = 12 \sqrt{(X'_{CR} - X'_{CS})^2 + (Y'_{CR} - Y'_{CS})^2} \quad \text{inches}$$

$$2.0 \quad \Delta\psi = \frac{(\psi_R - \psi_S)}{57.3} \quad \text{radians}$$

$$3.0 \quad \gamma_s = \arctan \left(\frac{Y'_{CR} - Y'_{CS}}{X'_{CR} - X'_{CS}} \right)$$

4.0 If $0.02 < |\Delta\psi|$ and $\theta < 1.000$ **GO TO 10.0**

$$5.0 \quad PR = \frac{|\Delta\psi|(a+b)}{2S} \quad (\text{path ratio}).$$

$$6.0 \quad \phi_\psi = \begin{cases} 0.78(PR) - 0.16(PR)^2, & \text{for } PR < 1.50 \\ 0.80, & \text{for } 1.50 \leq PR \end{cases}$$

$$7.0 \quad \phi_v = \begin{cases} 1.00 - 0.10(PR) - 0.28(PR)^2, & \text{for } PR < 1.50 \\ 0.20, & \text{for } 1.50 \leq PR \end{cases}$$

$$8.0 \quad \dot{\psi}_S = 57.3 \left\{ \sqrt{\left(\frac{\phi_\psi(a+b)\mu g}{k^2} \right) (|\Delta\psi|)} \right\} \text{sgn}(\Delta\psi) \quad \text{deg/sec}$$

$$9.0 \quad \dot{S} = \sqrt{2\phi_v\theta\mu g S} \quad \text{inches/sec}$$

GO TO 12.0

$$10.0 \quad \dot{\psi}_S = 57.3 \left\{ \sqrt{\frac{\mu g (\Delta \psi)^2}{\left[\frac{k^2 (1 - \theta)}{a + b} \right] |\Delta \psi| + \frac{S}{1.70}}} \right\} \text{sign}\{\Delta \psi\} \text{ deg/sec}$$

$$11.0 \quad \dot{S} = 1.70 \left[\frac{57.3 \mu g (\Delta \psi)}{\dot{\psi}_S} - \frac{k^2 (1 - \theta) |\dot{\psi}_S|}{57.3 (a + b)} \right] \text{ inches/sec}$$

$$12.0 \quad u_S = \dot{S} \cos(\gamma_S - \psi_S) \text{ inches/sec}$$

$$13.0 \quad v_S = \dot{S} \sin(\gamma_S - \psi_S) \text{ inches/sec}$$

$$14.0 \quad \text{Return with starting values: } u_S, v_S \text{ inches/sec} \\ \dot{\psi}_S \text{ degrees/sec}$$

Note that the above calculations can be simplified by means of the following steps:

Let

$$Q = \sqrt{\frac{\mu g}{\frac{k^2 (1 - \theta) |\Delta \psi|}{(a + b)} + \frac{S}{1.7}}} \quad (7)$$

Then

$$\dot{\psi}_S = \Delta \psi * Q * 57.3 \text{ deg/sec} \quad (8)$$

$$\dot{S} = S * Q \text{ inches/sec} \quad (9)$$

Several possible shortcomings in the cited technique for the generalized case have been investigated (see **Reference 23**). This technique should be considered a first approximation technique for the estimation of separation conditions which will produce better approximations than conventional constant deceleration techniques

(i.e., $V^2 = 2as$).