Yaw Inertia

In plane motion analyses of vehicle dynamics that include rotations, the moment of inertia of each vehicle about a vertical axis through its center of gravity is required.

The moment of inertia of a rigid body in rotational motion is analogous to its mass in translation. It is equal to the sum of the products of each mass particle of the body, m, multiplied by the square of the respective distance, r, from the axis of rotation.

$$I = \Sigma m r^2 \qquad (1)$$

The moment of inertia is generally given in one of the units listed in **Table 4**. (Note that the SMAC/EDSMAC computer programs use the units of LB-SEC^2-IN.)

Table 4 <u>Moments of Inertia</u> (Units and Conversion Factors)

Multiply	By	To Obtain
Slug-Feet^2	**1.0000**	LB-SEC^2-FT
	12.000	LB-SEC^2-IN
	0.13832	Kilopond-Meter-Sec^2
	1.35691	Kilogram-Meter^2
Kilogram-Meter^2	8.84362	LB-SEC^2-IN
	9.810	Kilopond-Meter-Sec^2
	0.73697	Slug-Feet^2
LB-SEC^2-IN	0.011527	Kilopond-Meter-Sec^2
	0.11308	Kilogram-Meter^2
	0.08333	Slug-Feet^2
Kilopond-Meter-Sec^2	86.75595	LB-SEC^2-IN
	0.10194	Kilogram-Meter^2
	7.22966	Slug-Feet^2

If the moment of inertia as defined by equation (1) is divided by the total mass of the body, the square of an average r, called the radius of gyration, is obtained. Thus, the radius of gyration is defined as the distance from the axis of rotation at which the whole mass of the body can be assumed to be concentrated for purposes of rotation.

A relatively simple approach to approximating the yaw inertia of a motor vehicle is presented in **Reference 26**. It is calculated as through the car were a rectangular block of uniform density:

$$I = M\frac{\left(a^2 + b^2\right)}{12} \quad , k^2 = \frac{(a^2 + b^2)}{12} \qquad (2)$$

where a and b are the length and breadth and

M is the mass of the car.

k^2 = radius of gyration squared

In **References 27 and 28**, approximation procedures for yaw inertia are presented which are based on measurements made under the condition of curb weight. The fitted linear functions of curb weight are displayed graphically in **Figure 24**.

For convenience in applications to automobiles where the weights include occupants and/or limited cargo weights, the fitted relationships of **References 27 and 28** have been converted to yield the radius of gyration squared corresponding to a given curb weight. For simplicity, it is assumed that the radius of gyration remains approximately constant under normal conditions of occupancy and/or limited cargo loading. Thus, the total mass can be multiplied by the curb-weight based radius of gyration squared to approximate the moment of inertia.

The radius of gyration squared is approximated by the following relationships:

$$k^2 = 4433.0 - \frac{4,493,600.2}{W_c} \text{ IN}^2 \qquad (3)$$

(**Reference 27**)

$$k^2 = 4775.9 - \frac{5,591,980.8}{W_c} \text{ IN}^2 \qquad (4)$$

(**Reference 28**)

where W_c = Curb Weight, LBS

Sample Applications

In **Table 5**, approximate values obtained with Equations (1), (2) and (3) are compared with reported measurement results for a number of specific vehicles (**Reference 28**). While it is always desirable, where possible, to use measured values for vehicle properties, it may be seen from **Table 5** that equations (2) and (3) yield reasonably accurate approximations. The sensitivity of the overall reconstruction results to ± variations of the approximate values can, of course, be evaluated to determine any need for greater accuracy.

Further Reading

In 1997, MacInnis, et. al (**Reference 29**) in a presentation of a review of Moment of Inertia estimation techniques found that "the idealized vehicle prism method and "mab" rule of thumb have often yielded very good results".

- Equation (2) on the previous page is the technique identified in **Reference 29** as the "vehicle prism model".
- The "mab" estimation technique is referred to in the paper as "the most commonly cited estimation of the yaw moment". They cite Bastow (**Reference 30**) as a source for a description and derivation of the equation (they could not ascertain the original source for the equation).

A brief description of other sources for measurements of vehicle properties is as follows:

- Bixel (**Reference 31**) includes in Appendix 1 measurements of approx. 14 vehicles made at the VIMF (Vehicle Inertia Measurement Facility).
- Garrott (**Reference 32**) includes measurements made at the IPMD (Inertial Parameter Measurement Device) which is located at the VRTC (Vehicle Research and Test Center). Inertial data and/or CG height data from 356 measurement tests are included in the paper.
- Curzon (**Reference 33**) includes measured properties of some vehicles as well as a comparison of various estimation techniques.

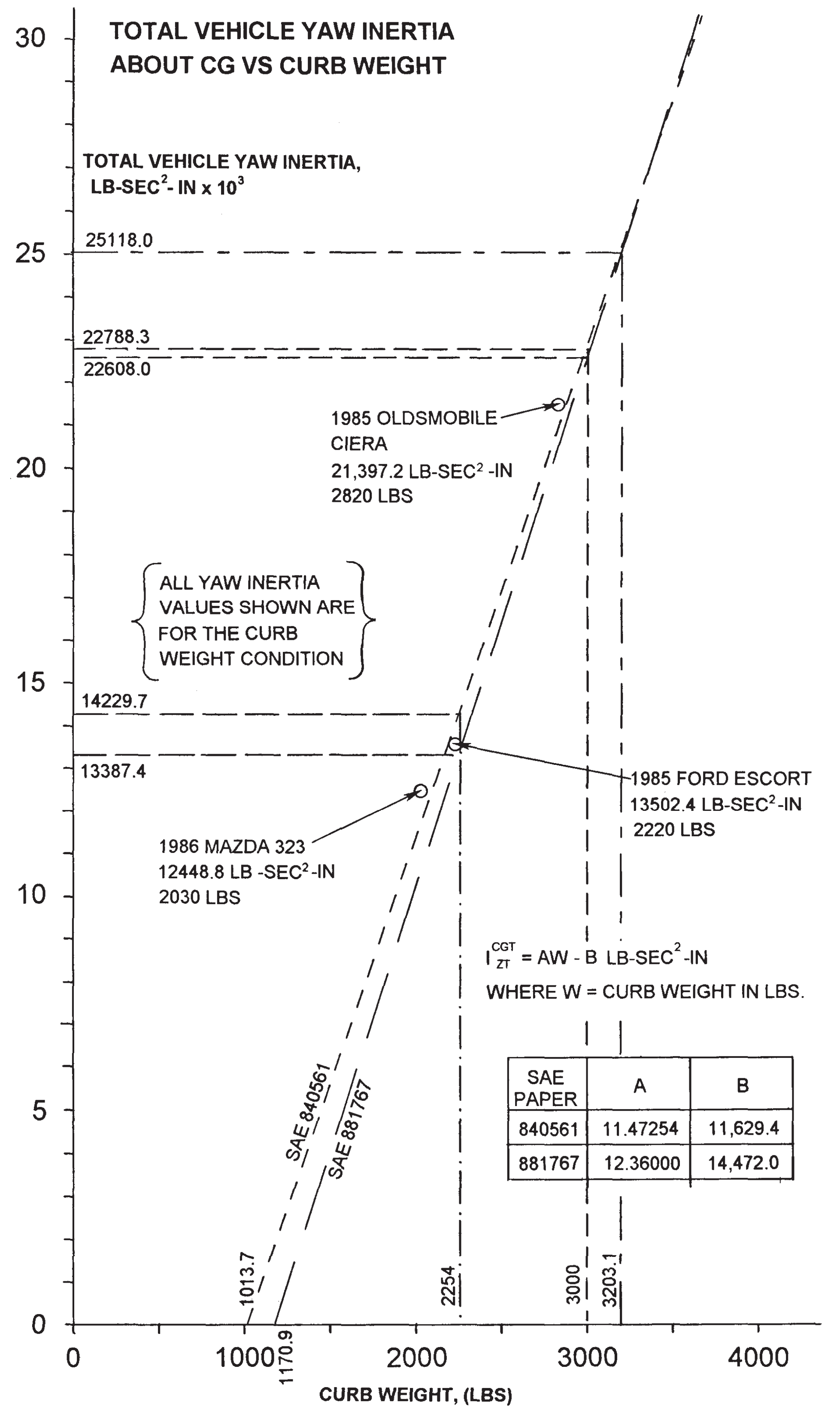

Figure 24 Total Vehicle Yaw Inertia About CG vs. Curb Weight

Table 5 Sample Application Results

Vehicle	*Curb Weight	**Overall Length	**Overall Width	*Measured Radius of Gyration Squared	Eq'n (2)	Eq'n (3)	Eq'n (4)
	LBS	*INCHES*	*INCHES*	*IN^2*	*IN^2*	*IN^2*	*IN^2*
'86 Mazda 323	2030.	161.8	65.0	2369.6	2533.7 +6.9%	2219.4 -6.3%	2021.2 -14.7%
'86 Hyundai Excel	2070.	161.0	62.6	2379.6	2486.6 +4.5%	2262.2 -4.9%	2074.5 -12.8%
'87 Nissan Sentra	2140.	168.9	64.6	2344.8	2725.0 +16.2%	2333.2 -0.5%	2162.8 -7.8%
'87 Toyota Corolla	2196.	166.5	64.6	2493.9	2658.0 +6.6%	2386.7 -4.3%	2229.5 -10.6%
'86 Ford Escort	2290.	163.8	65.7	2278.3	2595.6 +13.9%	2470.7 +8.4%	2334.0 +2.4%
'83 Toyota Camry	2462.	175.6	66.5	2840.8	2938.1 +3.4%	2607.8 -8.2%	2504.6 -11.8%
'87 Chrysler LeBaron	2690.	179.1	68.1	2694.3	3059.5 +13.6%	2762.5 +2.5%	2697.1 +0.1%
'87 Mercedes 190	2870.	175.2	66.1	2557.5	2922.0 +14.3%	2867.3 +12.1%	2827.5 +10.6%
'84 Mercury Marquis	3860.	196.5	70.9	3476.0	3636.6 +4.6%	3268.9 -6.0%	3327.2 -4.3%
'87 Ford Thunderbird	3920.	202.0	71.3	2922.2	+3824.0 +30.9%	3286.7 +12.5%	3349.4 +14.6%
'80 Oldsmobile 98	4164.	218.9	74.8	4111.4	4459.4 +8.5%	3353.8 -18.4%	3433.0 -16.5%

*Reference 3, Curb Wgts., as equipped; Moment of Inertia measured.
** Canadian Vehicle Specifications (1998)

22 Burns, J. C.,. Meyyer, W. E. , Hayhoe, G. F., Ivey, D. L., "**Friction Variations**", Chapter 5 of ***The Influence of Roadway Surface Discontinuities on Safety***, Transportation Research Board, Natural Research Council, 1984.

23 McHenry, R.R., McHenry, B.G., **"National Crash Severity Study - Quality Control - Task V: Analysis to Refine Spinout Aspects of CRASH",** Contract DOT-HS-6-01442, January 1981

24 Marquard, E., **"Progress in the calculations of Vehicle Collisions",** Automobiletechnische Zeitschrift, Jarq. 68, Heft 3, 1966

25 McHenry, R.R., **"User's Manual for the Crash Computer Program, Part 2 - Final Report",** Calspan Report No. ZQ-5708-V-3, Contract No. DOT-HS-5-01124, January 1976.

26 Grime, G., Jones, I.S., **"Car Collisions – The Movement of Cars and Their Occupants in Accidents",** Proceedings of the Institution of Mechanical Engineers, Volume 184, Part 2A, Number 5, 1969-70.

27 Riede, P. M., Leffert, R.L., Cobb, W.A.,**"Typical Vehicle Parameters for Dynamic Studies Revised for the 1980's",** SAE paper 840561.

28 Garrott, W.R., Monk, M.W., Chrstos, J.P., **"Vehicle Inertial Parameters – Measured Values and Approximations",** SAE paper 881767

29 MacInnis, D.D., Cliff, W.E., Ising, K.W, **"A Comparison of Moment of Inertia Estimation Techniques for Vehicle Dynamics Simulations",** SAE paper 970951

30 Bastow, Donald, **Car Suspension and Handling**, Pentech Press, London 1993

31 Bixel, A.B., Heydinger, G.J., Durisek, N.J., Guenther, D.A., Novak, S.J., **"Development in Vehicle Center of Gravity and Inertial Parameter Estimation and Measurement",** SAE paper 950356

32 Garrott, W.R., **"Measured Vehicle Inertial Parameters – NHTSA's Data Through September 1992",** SAE paper 930897

33 Curzon, A.M., Cooperrider, N.K., Limbert, D.A., **"Light Truck Inertial Properties",** SAE paper 910122

34 McHenry, R.R., **"Automotive Crash Injury Research "Estimated Traveling Speed and Fatalities"**, Cornell Aeronautical Laboratory Report No. VJ-2721-R1, Jan 1969

35 McHenry, R.R., **"Tri-Level Accident Research Study",** Calspan Report No. ZM-5086-V-2, May 1973

36 Emori, R.I., **"Analytical Approach to Automobile Collisions",** SAE paper 68-0016

37 Mason, R.P., Whitcomb, D.W. , **"The Estimation of Accident Impact Speed",** Cornell Aeronautical Laboratory Report No. YB-3109-V-1, August 1972

38 Campbell, K.L., **"Energy Basis for Collision Severity",** SAE paper 74-0565, 3rd International Conference on Occupant Protection, Troy, Michigan, July 10-12, 1974.

39 McHenry, R.R., Deleys, N.J., **"Determination of Physical Criteria for Energy Conversion System"**, Bureau of Public Roads, Program Review Meeting of R&D of Traffic Systems, Gaithersburg, Maryland, December 6-8, 1966.

40 McHenry, R.R.,**"Yielding-Barrier Test Data Base - Refinement of Damage Data Tables in the Crash Program"**, Calspan Report No. ZR-5954-V-1, DOT-HS-5-01124December 1976.

41 Smith, R.A, Noga, J.T. , **"Accuracy and Sensitivity of CRASH"**, SAE paper 82-1169.

42 McHenry, R.R., J.P.Lynch, D.J.Segal.,**"Research Input for Computer Simulation of Automobile Collisions"**, Contract DOT-HS-7-01511, Calspan Corporation Report No. ZQ-6057-V-1,June 1977

43 Monk, MW, Guenther, D., **"Update of CRASH2 Computer Model Damage Tables",** DOT-HS-806446, March 1983

44 McHenry, R.R., **"User's Manual for the Crash Computer Program, Part 2 - Final Report"**, Calspan Report No. ZQ-5708-V-3, Contract No. DOT-HS-5-01124, January 1976.

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Accident Reconstruction

2008

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