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Effects of Restitution in the Application of Crush Coefficients

Raymond R. McHenry and Brian G. McHenry
McHenry Consultants, Inc.
Cary, NC

ABSTRACT

Effects of restitution on damage interpretations are compounded by the fact that restitution acts to reduce the amount of residual deformation, for a given maximum dynamic crush, while also acting to increase the total impact speed change. This paper presents a revised analytical procedure to include restitution effects for the CRASH program and refinements to the restitution modeling within the SMAC program. The conversion of vehicle impact test results into inputs for the two revised programs is also included. The effects of the refinements to the damage analysis procedures on reconstruction results are illustrated by direct comparisons with corresponding results produced by the original SMAC and CRASH programs and with measured data from full scale vehicle impact tests.

INTRODUCTION

During a motor vehicle collision, the maximum dynamic deformation generally exceeds the residual deformation. Subsequent to the peak dynamic deformation, the collision partners begin a restitution phase as the deformed structures restore kinetic energy, or “spring” back. The restitution force level and duration determine the impulse that acts on the collision partners during the restitution phase.

When an accident vehicle is examined, the residual, or permanent, deformation is observed and/or measured. The original form of damage analysis in CRASH does not include provisions for the effects of restitution. The original SMAC collision routine includes a simplified restitution model which is cumbersome to apply, can be sensitive to time increment size, and tends to over-predict the residual damage. The resulting effects on the accuracy of damage-based reconstructed values of ΔV, for the case of direct, central barrier collisions, ranges from approximately 10 to 30% underestimates, depending on properties of the specific vehicle and the extent of residual crush. For the case of oblique, non-central collisions, a similar range of effects is anticipated on the basis of indirect measures of corresponding restitution values [**1, 2**][1].

At the present time, crush coefficients for vehicle collision analysis are predominantly based on impact speeds and damage measurements from rigid, fixed barrier crash tests. The residual damage is correlated with the impact speed by means of fitted linear relationships. In general, there is no consideration given to the effects of restitution in applications of the fitted crush coefficients. However, the ignored effects of restitution on the total impact speed-change, corresponding to a given amount of residual crush, are compounded by the fact that restitution acts to reduce the amount of residual deformation, for a given maximum dynamic crush, while also acting to increase the total impact speed change. Thus, substantially different vehicles can share nearly equal slopes and intercepts in CRASH-type plots of the approach period speed-change as a function of residual crush. This can occur even though the actual exposure severity for a given residual crush may be significantly different.

The effects of restitution on damage interpretations became an important topic of interest in relation to recent efforts by McHenry Consultants, Inc. aimed at (a) reducing the sensitivity of restitution control in the original SMAC computer program [**3**] and (b) achieving significant improvements in the general accuracy level of damage interpretations. The related modifications of the SMAC and CRASH computer programs have retained the existing forms of the fitted crush coefficients. However, supplementary information regarding the restitution behavior of individual vehicles serves as the basis for refinements in the damage analysis aspects of the cited computer programs.

ANALYTICAL APPROACH

The analytical developments presented in this paper are based on limited test data that are available on the restitution behavior of automobiles in direct central collisions against rigid barriers. It is assumed herein that an analytical model of the unit-width structural properties, in terms of the load-deflection characteristics during loading and unloading, that adequately correlates with the results of direct central

1 Numbers in brackets [ ] indicate references at end of paper

collision tests, will also produce a reasonable approximation of restitution behavior in oblique, non-central collisions.

Direct measures of restitution in oblique, non-central collisions of automobiles are not known to be readily available. However, analytical procedures have been developed (e.g., Brach [**1**], Ishikawa [**2**]) to extract approximations of the effective coefficient of restitution from such test data. These approaches may provide a means of extending the current investigation to include comparisons with oblique, non-central collisions.

The assumed form of crush behavior, including restitution, is depicted in Figure 1A. The objective of the analytical developments has been to establish relationships among the variables depicted in Figure 1A and the existing CRASH coefficients A and B [**4**] that will **maintain the linear relationship depicted in Figure 1B**. On the basis of results of the analysis, it is concluded that the linear relationship depicted in Figure 1B can be maintained without changes in A and B, over wide ranges of properties of the form depicted in Figure 1A. Thus, the relationship of Figure 1B does not uniquely define a combination of crush and restitution properties. Restitution consists of two separate aspects: (1) a partial dimensional recovery and (2) a partial restoration of kinetic energy. The specific combination of the two restitution aspects is determined by three measures of restitution behavior from test data that serve as the basis for four fitted constants (K_1, K_2, ρ, Γ, see Appendices). The crush resistance during increasing loads, K_1, is determined by CRASH coefficients A and B and by the extent of dimensional recovery. The unit-width crush resistance during unloading, K_2, is determined by K_1 and the extent of restoration of kinetic energy. The nature and extent of effects on damage interpretations are outlined in the following paragraphs:

In Figures 2 and 3, full-frontal crush properties are depicted for two substantially different equal-width hypothetical vehicles which share identical crush coefficients (i.e., A and B in the CRASH format). Yet, at 20 inches of residual crush, the total impact speed-changes of the two vehicles differ by approximately 23% (i.e., approximately 7.3 MPH) as shown in Figure 4 and 5.

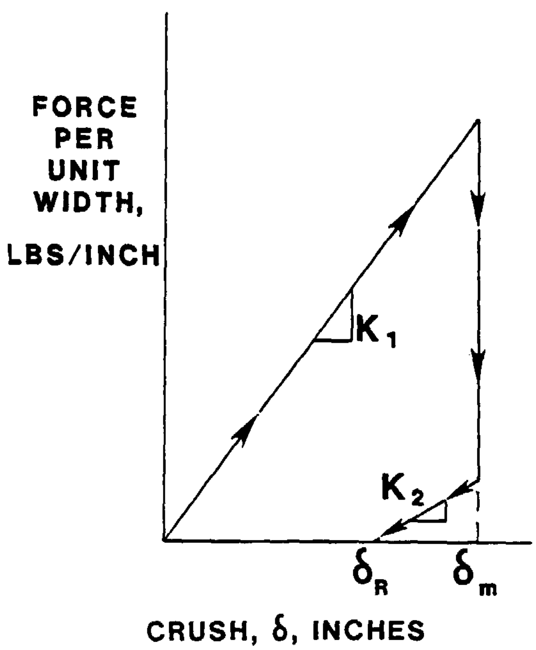


Figure 1A: Force v. Crush

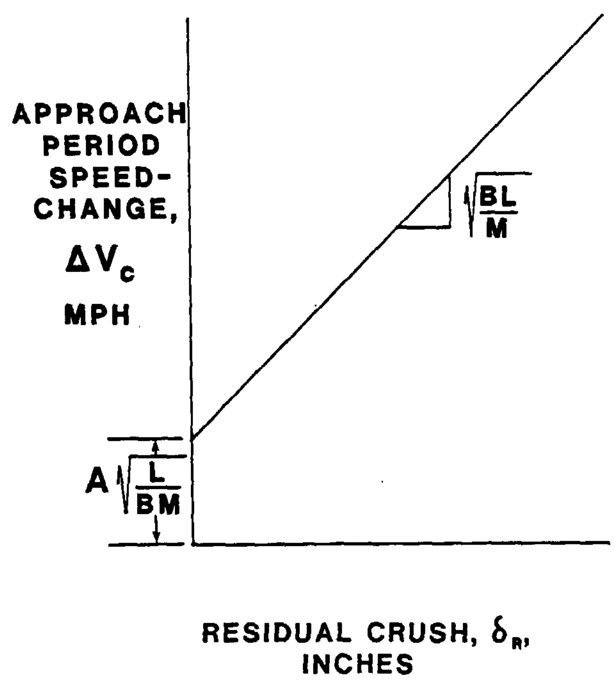


Figure 1B: Approach-Period Speed-Change vs. Residual Crush, Symmetrical Central Impacts

Figure 1 Assumed Form of Crush Behavior

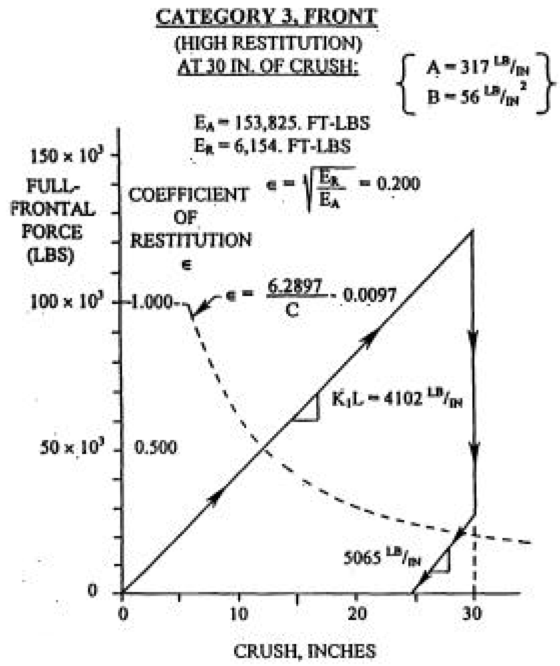


Figure 2 Full-Frontal Crush Properties

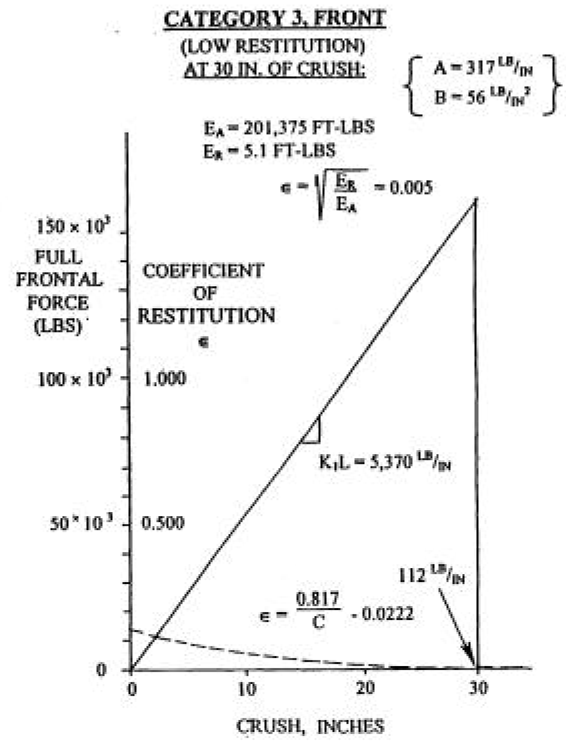


Figure 3 Full-Frontal Crush Properties

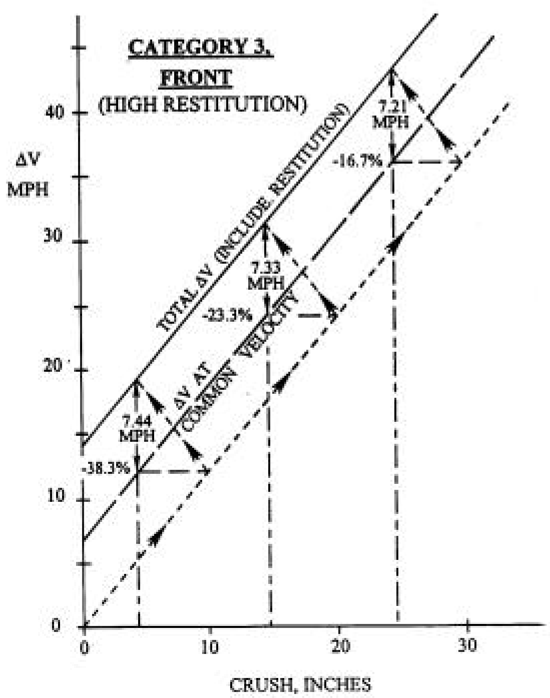


Figure 4 Impact Speed Change v. Crush, Category 3 Frontal, High Restitution

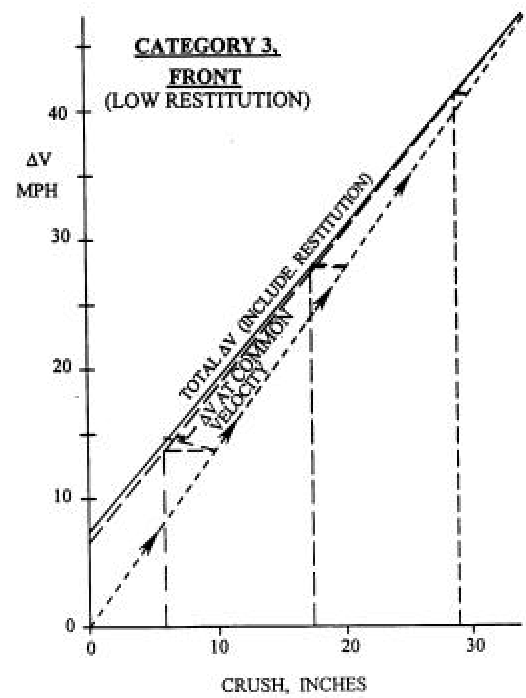


Figure 5 Impact Speed-Change v. Crush, Category 3 Frontal, Low Restitution

The vehicle properties depicted in Figures 2 and 3 are based on theoretical requirements to achieve a linear relationship between ΔV_c (i.e., the impact speed-change during the approach period) and the residual crush. To the extent that such a linear relationship constitutes a reasonable approximation of actual behavior, the underlying crush resistance and restitution definitions of Figures 2 and 3 are supported. It should be noted that the low-restitution vehicle of Figure 3 is approximately 31% stiffer than the high-restitution vehicle of Figure 2. Yet the crash behavior of each vehicle is defined by identical values of the current crush coefficients:

per reference [**5**]:

$$A = 317 \text{ LB/IN}$$

$$B = 56 \text{ LB/IN}^2$$

per reference [**6**]:

$$b_0 = 6.741 \text{ MPH}$$

$$b_1 = 1.191 \text{ MPH/IN}$$

In view of the significantly different values for the total ΔV at given values of residual crush as demonstrated in Figures 4 and 5, it is obvious that the current crush coefficients must be supplemented with restitution information. It should be noted that an absence of restitution throughout the entire range of deformation would produce zero values for the fitted coefficients A and b_0.

COMPARISON OF RESTITUTION IN ORIGINAL AND REVISED SMAC

At the time of development of the original SMAC computer program, (1972, Reference [**3**]) emphasis was on demonstration of the feasibility of the overall concept. The selected analytical approach to restitution at that time (presently retained in both original SMAC and EDSMAC) included the use of extensive simplifying assumptions aimed at reducing the requirements for associated computer memory and logic. In particular, identical load-deflection rates were applied for loading and unloading of the individual radial vectors, that define the collision interface, and the unloading was implemented at force levels close to the peak values [**7**]. As a part of the selected simulation approach, the “unloaded” lengths of the individual vectors were adjusted at each time increment during increases in loading. The associated coding thereby avoided any needs for logic to detect the end of loading at the individual radial vectors and for additional memory to store related information. The selected original approach permits reasonable control of restitution over a limited operating range about a given value of deformation for which restitution information is available. However, the high load level during restitution makes the simulation excessively sensitive to time-increment size. It is also cumbersome to apply and it tends to over-predict the residual damage.

Since the early 1990’s the extent of limitations on logic and memory, that are imposed by readily available computers, are substantially reduced. Also, the available data on restitution behavior have been increased. Therefore, the collision routine of SMAC has been modified to achieve improvements in reconstruction accuracy. Results representative of the changes in restitution and in the ratio of residual to maximum dynamic crush, each as a function of the maximum dynamic crush and the residual crush, are depicted in Figures 6A and 6B, respectively

The ratio, δ_f/δ_m, clearly must vary from zero, at zero residual crush, to 1.000 at that value of residual crush where there is no further restitution. Figure 6B shows that original SMAC (EDSMAC) does not vary the ratio of δ_f/δ_m as a function of δ_f in a realistic manner.

It is obvious from Figures 6A and 6B that the ratio of (δ_f/δ_m) from the original SMAC program (or the equivalent ratio (C_R/C_T) in [**8**]) should not be used as a basis for determining the appropriate K_V value for the SMAC program corresponding to the fitted CRASH coefficients A and B. If a measured value of $(\delta_f/\delta_m)_1$ for a given maximum dynamic crush value, $(\delta_m)_1$ is available from a crash test, equation (8) in Appendix 1 can serve to define the appropriate K_v for given values of A and B.

Implementation of a revised damage analysis procedure, that includes restitution effects, was outlined for the CRASH computer program in [**9**]. A corresponding revision, which was recently incorporated in the SMAC computer program, has been applied to generate the responses presented on the following pages. Detailed analytical relationships are presented in the Appendices.

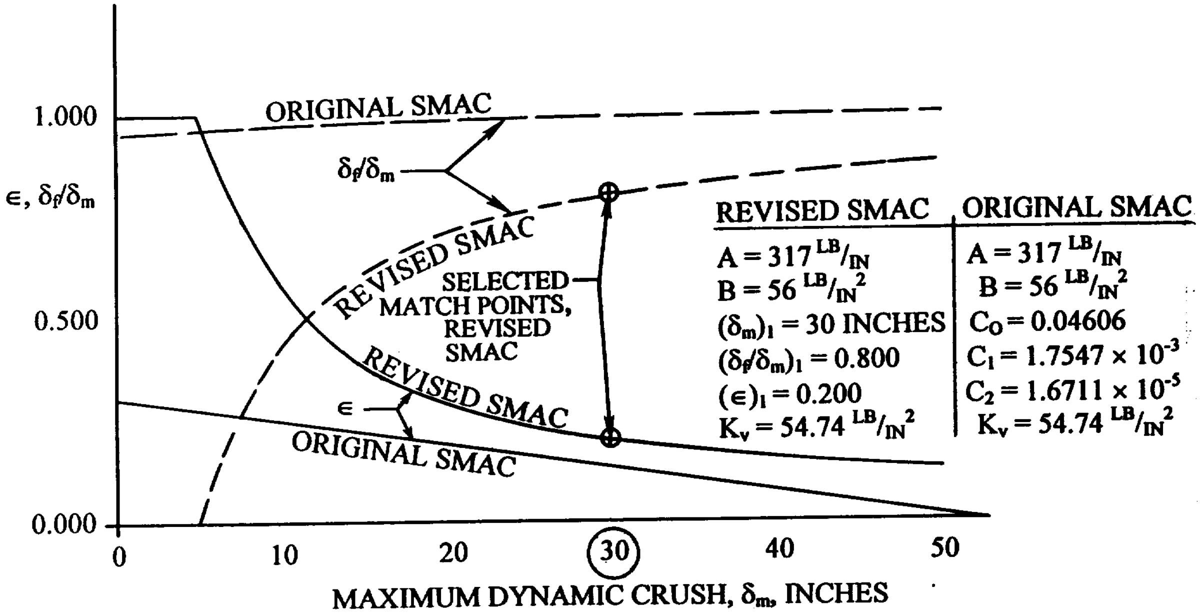


Figure 6A Plotted Against Maximum Dynamic Crush

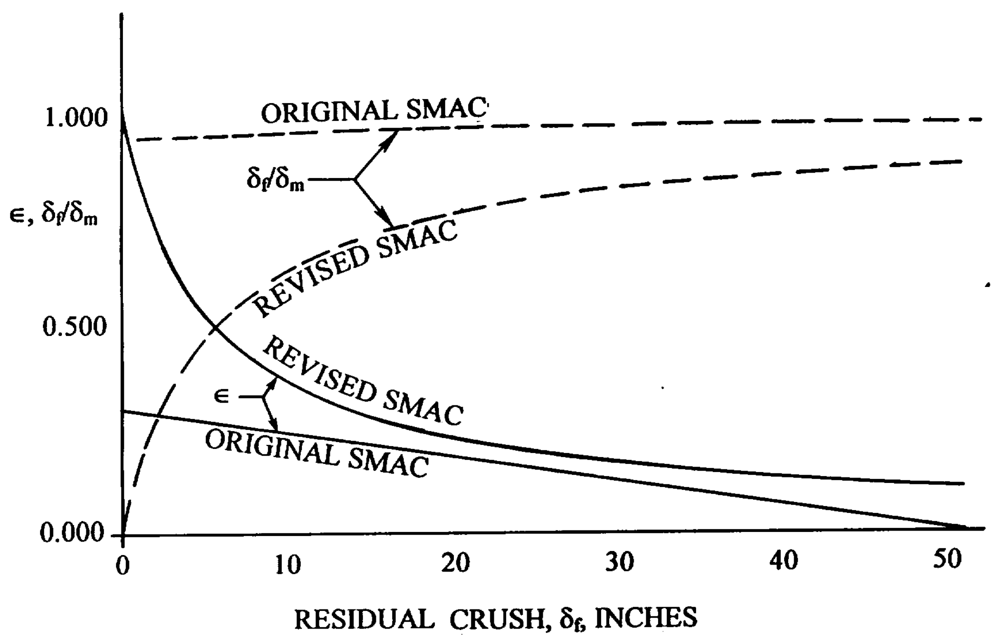


Figure 6B Plotted Against Residual Crush

Figure 6 Simulated Restitution Behavior in Original and Revised SMAC Routines

RESULTS

Damage-Based Reconstructions of Delta-V (DV)

In the following comparisons of (1) reconstruction results obtained with the original CRASH (EDCRASH) program and the revised CRASH program and also (2) reconstruction results obtained with the original SMAC (EDSMAC) program and the revised SMAC program, the differences in damage-based ΔV values are produced entirely by the inclusion of realistic restitution properties in a fully defined hypothetical vehicle. Neglect of restitution effects constitutes an analytical error since it involves the total omission of a significant aspect of the collision, particularly at impact speed changes below 30 mph.

In the application of any reconstruction technique to physical evidence there are many potential sources of error which must be properly taken into account. The presented comparisons isolate the changes in reconstruction results that are produced by effects of restitution.

It should be noted that meaningful direct comparisons of theoretical results with test data require that the accuracy and the repeatability of the test data be established. Also, such comparisons yield the total error from the combined sources.

A comparison of results obtained with the original CRASH program and the revised CRASH program for a defined hypothetical vehicle is presented in Figure 7. The figure illustrates the fact that an application of the original CRASH damage analysis procedure can produce ΔV errors (i.e., underestimates) in the range of 10-30%.

A comparison of results obtained with the NHTSA SMAC (EDSMAC) program and the revised SMAC program for the same hypothetical vehicle is presented in Figure 8. The figure illustrates the fact that an application of the NHTSA SMAC (EDSMAC) program for damage analysis purposes can also produce ΔV errors (i.e., underestimates) in the range of 10-30%.

The original form of restitution control in the SMAC program acts to return part of the absorbed energy but it does so with a less-than-actual dimensional recovery (Figure 9**,** Figure 10 **).** As a result, a purely damage-based determination of ΔV by means of the original SMAC program tends to underestimate the true value of ΔV due to the fact that the residual damage for a given ΔV is overestimated. The cited error source in damage interpretations in original SMAC has led to some misguided adjustments in the crush stiffness (e.g., [**10**]) to achieve a match of predicted damage extent.

A comparison of the results obtained by the various versions of SMAC and CRASH are combined on Figure 11. It should be noted that the results for the revised versions of SMAC and CRASH are identical. The retention of an excessive amount of predicted residual damage by original SMAC makes the ΔV errors at low speeds somewhat greater than those of original CRASH.

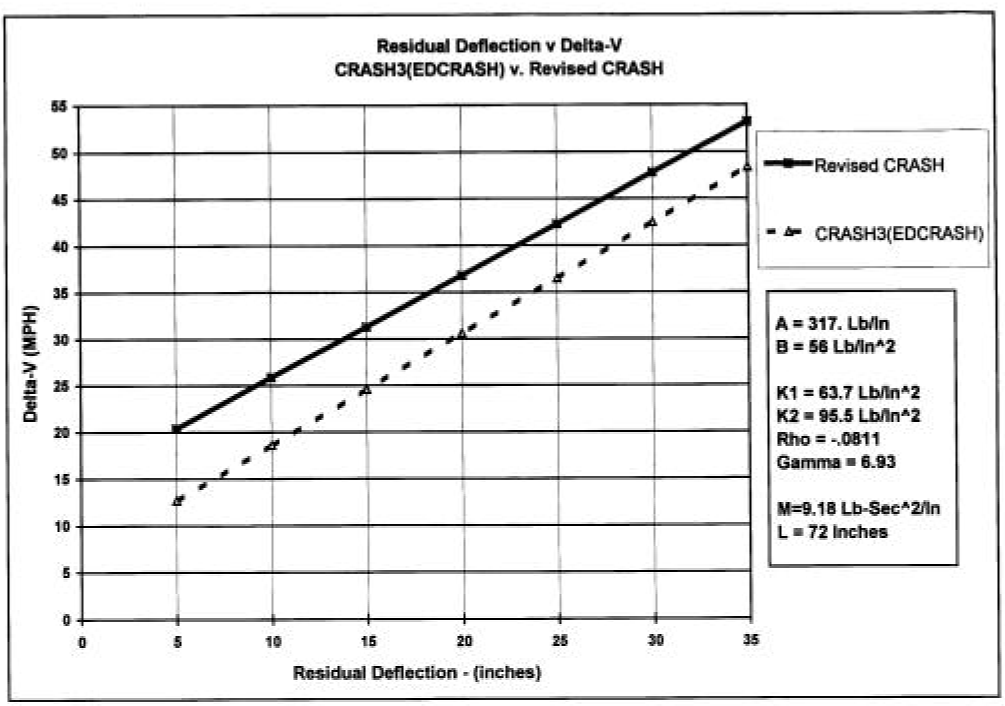


Figure 7 Comparison Residual Deflection Delta-V CRASH3 (EDCRASH) v. Revised CRASH

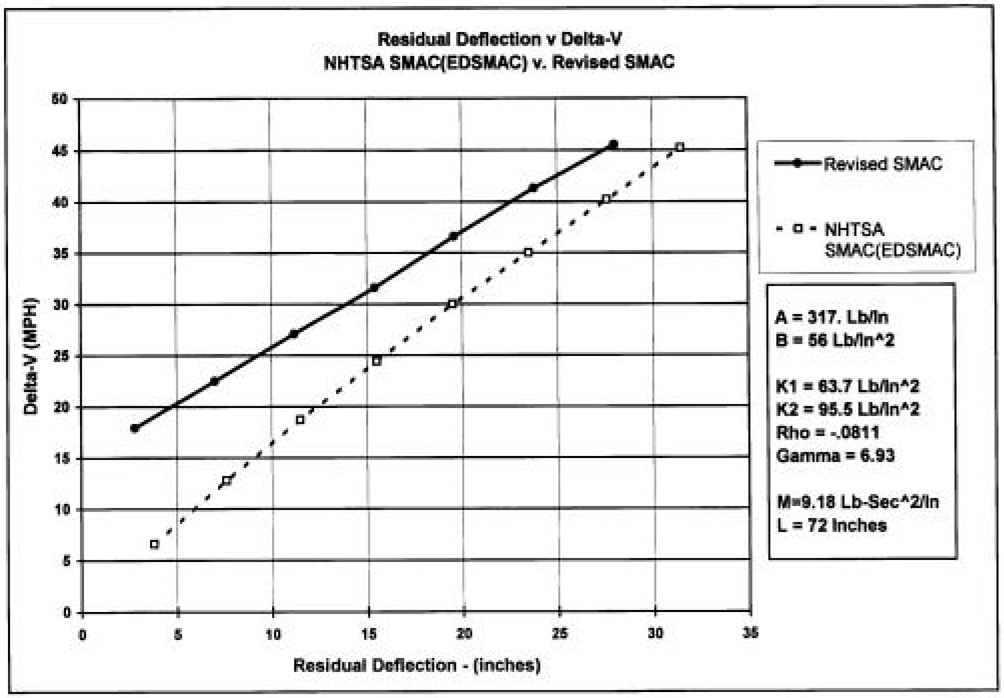


Figure 8 Residual Deflection v. DeltaV for Original SMAC (EDSMAC) v. Revised SMAC

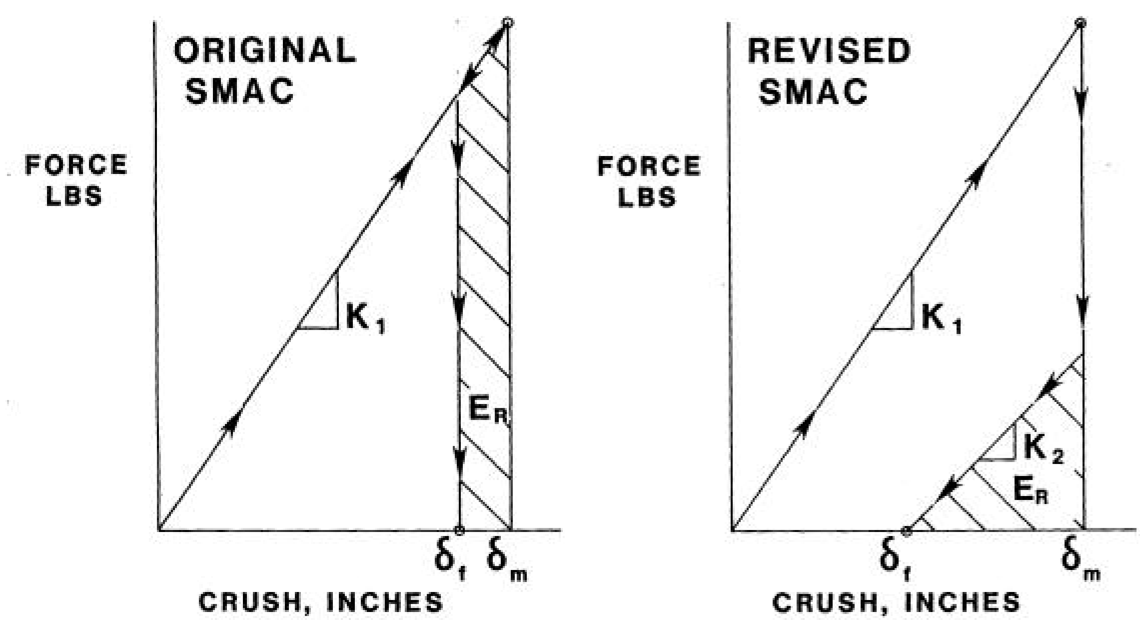


Figure 9 Simulated Force Deflection Characteristics for the original SMAC (EDSMAC) program and the Revised SMAC program

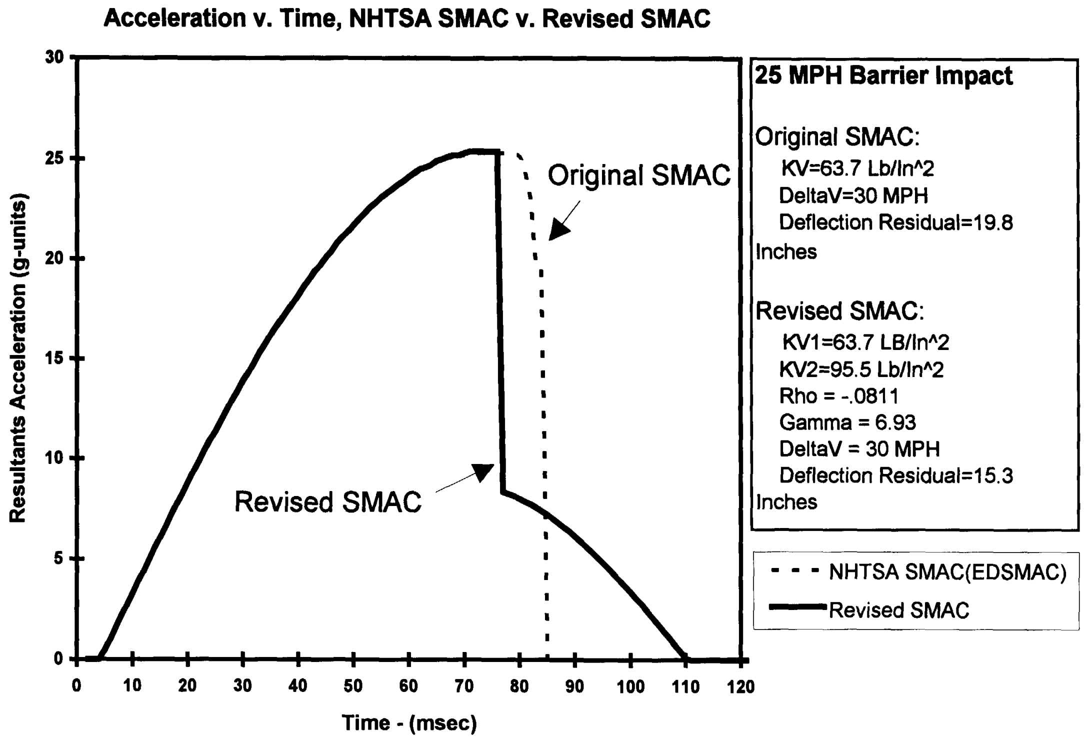


Figure 10 Acceleration v. Time for Original SMAC (EDSMAC) and Revised SMAC

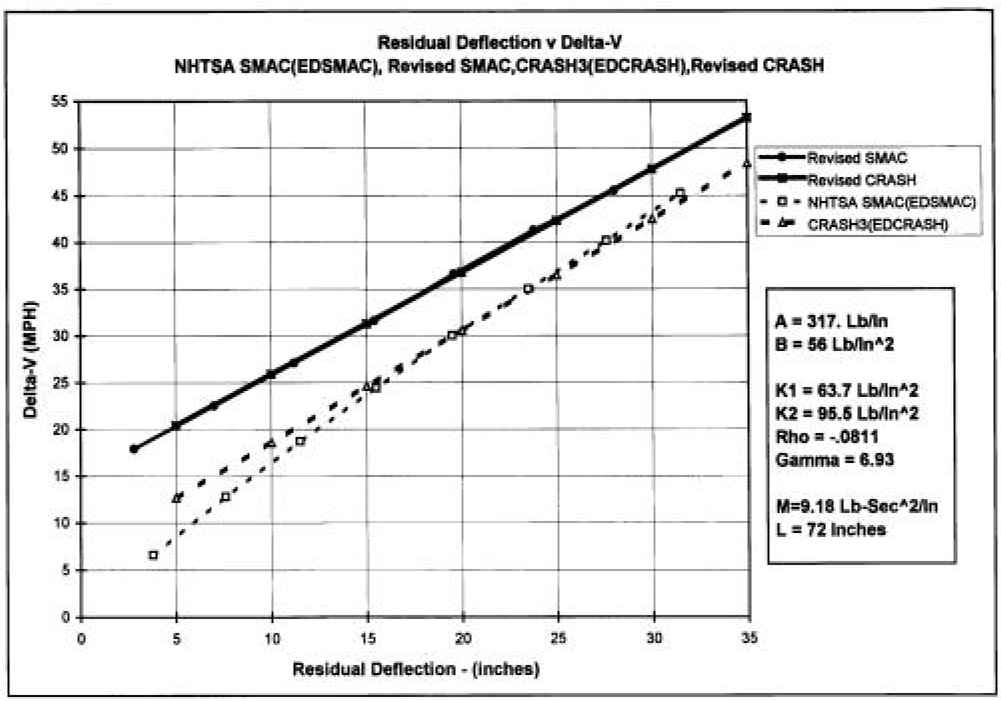


Figure 11 Residual Deflection v. DV for CRASH & SMAC program

Comparison of the Mathematical Model of Crush Behavior with Test Data

The lack of exact repeatability of full scale crash test results introduces "scatter" in comparisons of the detailed results of multiple tests. Measurements related to restitution behavior have been found to include an unusually large amount of scatter, particularly at low values of impact speed-changes. The effective coefficient of restitution can include significant effects of forces that are external to the two-body collision system. For example, in an SAE barrier crash, damaged running gear (e.g., jammed or impeded front wheels) can produce significant drag forces that act to reduce the rebound velocity, particularly at low levels of returned energy. Also, energy absorbers on bumpers can act to delay the return of a portion of the absorbed energy.

The mathematical model defined herein addresses the responses of the vehicle structure only. The magnitude of the effects of external forces on individual full-scale tests has not been measured. In the absence of special tests that measure the isolated responses of the structure only (e.g., tests run on casters), it is necessary to focus attention on correlation with the higher returned-energy end of the available measured responses.

While some comprehensive test data from individual tests are available, test series over a range of impact speeds with comprehensive reporting are still relatively rare.

In Figure 12, comprehensive test data from a series of four tests of '79 through '82 Ford LTDs [**11**] are compared with the fitted mathematical model. In Figure 13, test data for six '81-'85 Ford Escorts [**12**] which did not include reporting of the ratio of residual to maximum crush, are compared with the fitted mathematical model. In Figure 14, test data from a series of five tests on 1975-1979 VW Rabbits [**13**] which did not include reporting of the coefficient of restitution are compared with the fitted mathematical model.

Note that the measurements of different aspects of restitution behavior in crash tests that are presented in Figures 12, 13 and 14 include scatter produced at least in part by the previously cited effects.

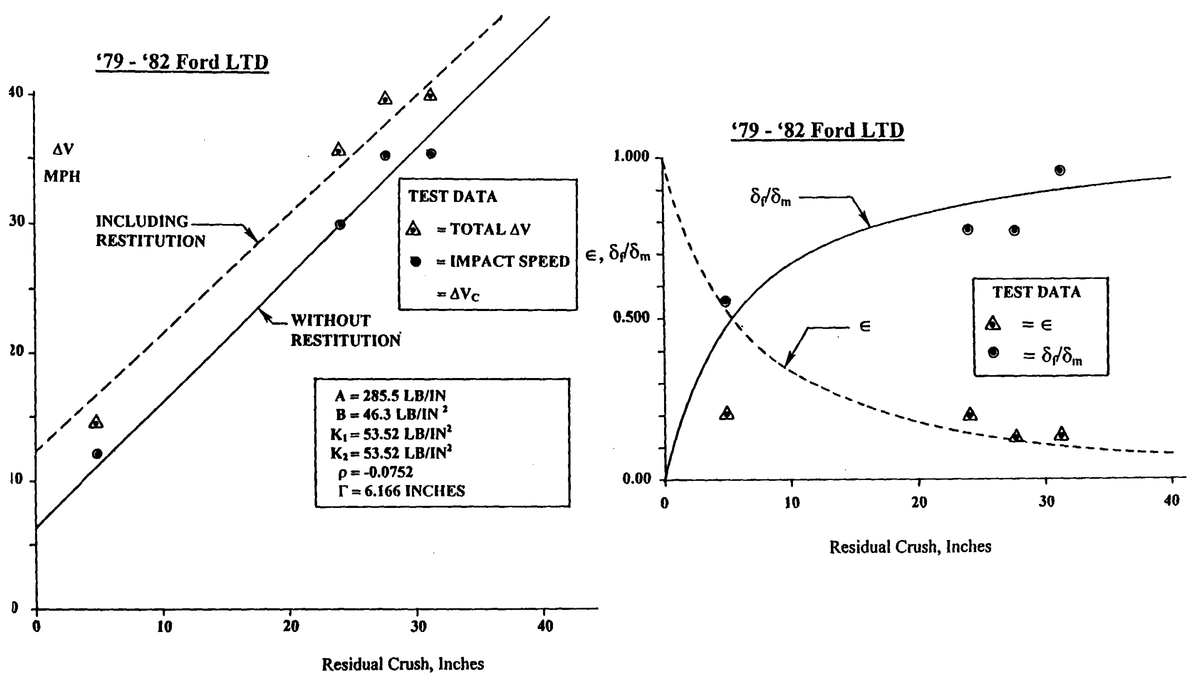


Figure 12 Comparison of Mathematical Model with Ford LTD Tests [11]

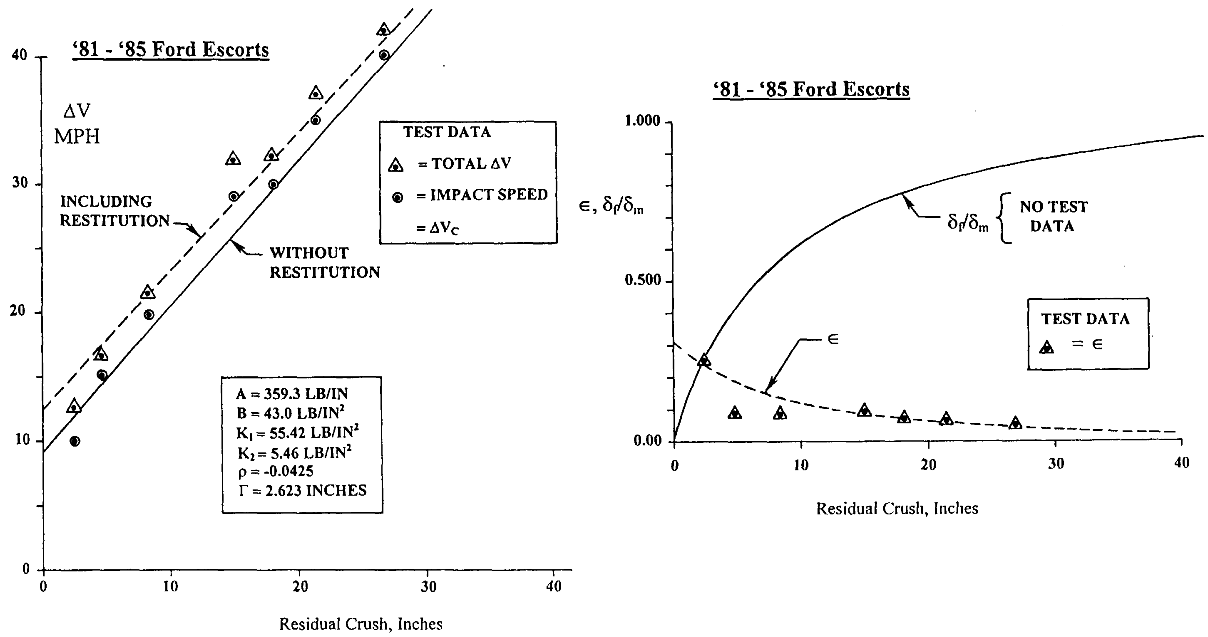


Figure 13 Comparison of Mathematical Model with Ford Escort Tests [12]

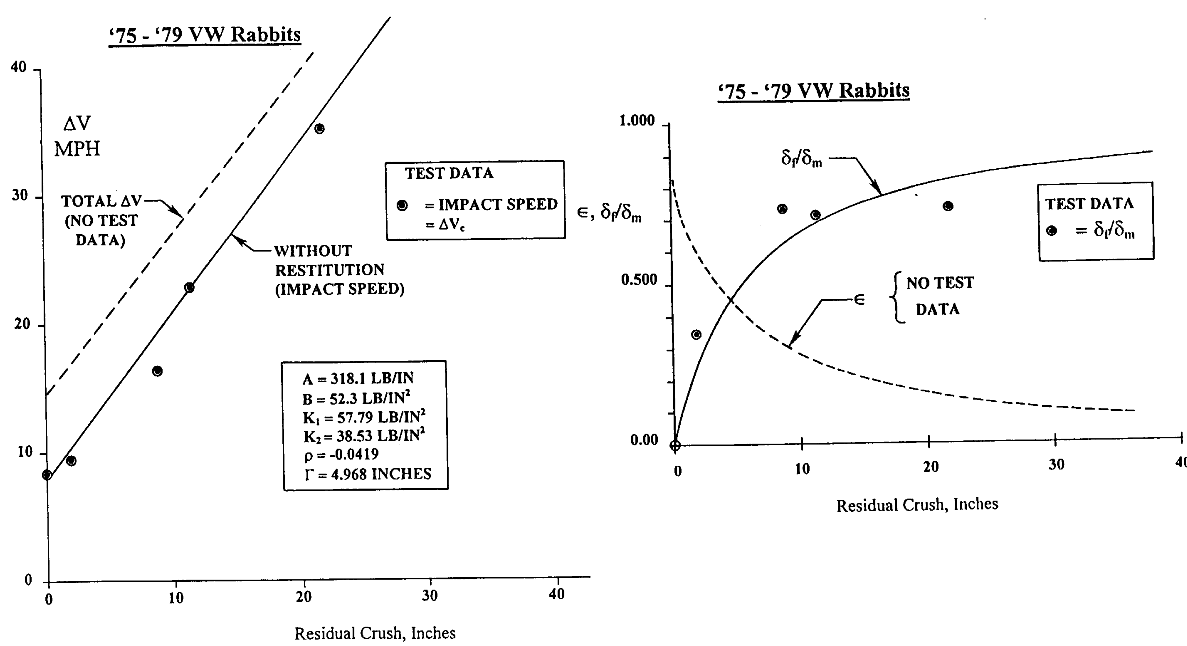
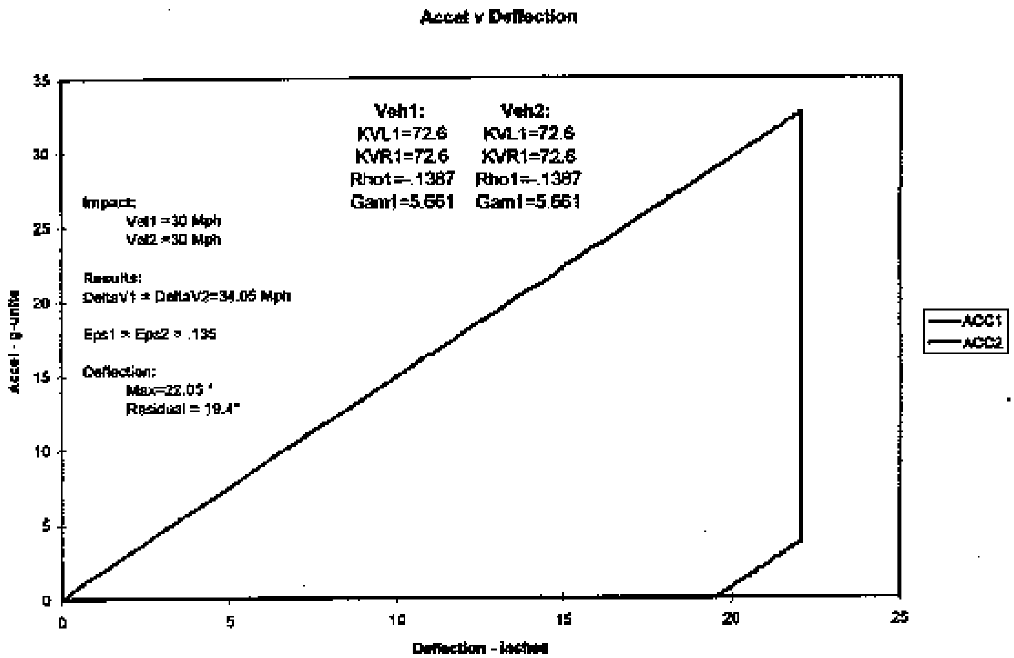


Figure 14 Comparison of Mathematical Model with VW Rabbit Tests [13]

Collisions between Vehicles with Different Restitution Properties

In Figure 15 the results of a 30 MPH head-on mirror image collision, simulated with the revised SMAC program, are displayed. Note that in the simulated collision of Figure 15, the restitution properties of the collision partners are identical. Figure 16 depicts the simulated results with the revised SMAC program of a head-on 27 MPH collision between a high restitution and a low restitution vehicle. Note that the effective restitution is an intermediate value.

In the SMAC implementation, force equilibrium is maintained between the interacting structures of the collision partners at all points within the contact zone throughout the unloading process. By this means, the two vehicles continue to interact during the unloading process until they each reach their residual values of crush at all contact points.



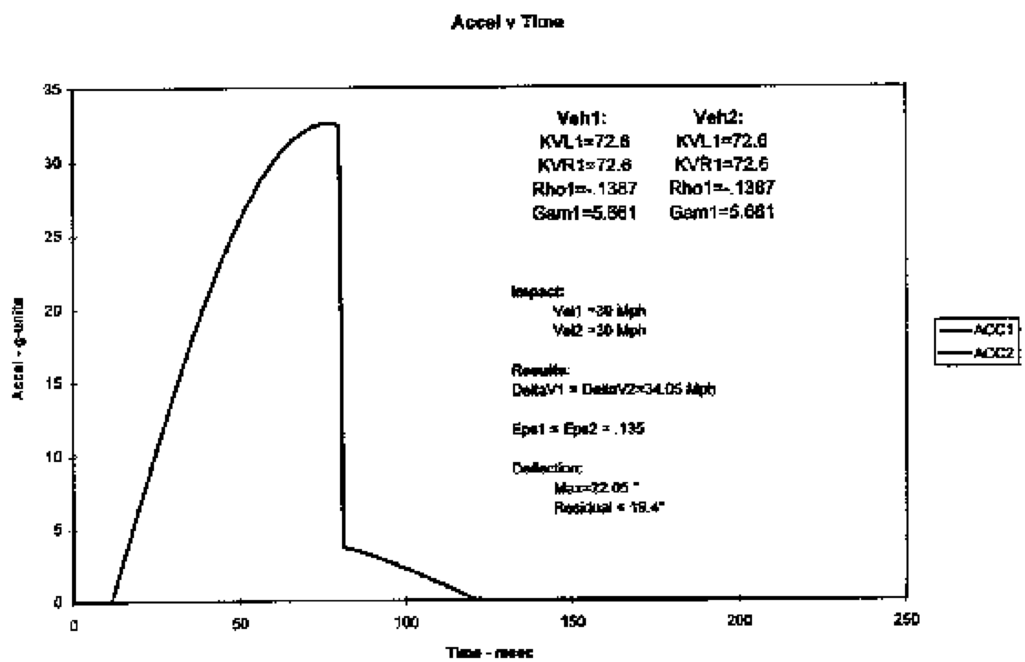


Figure 15 Example Response characteristics of revised SMAC collision routine, 30 MPH Head-On Mirror Image Vehicles

In the CRASH implementation [9], the restored energy for each of the collision partners is separately calculated by means of integrations across the damage interface. The resulting values are added together and then combined with the total absorbed energy for application in the calculation of ΔV_1 and ΔV_2.

In each form of implementation, the effective overall coefficient of restitution in a given collision includes effects of the width and location on each vehicle of the contact area, the detailed damage profiles, and the individual unit-width crush properties of the collision partners. This combination of effects is believed to constitute a realistic analytical representation of the actual physical system during the unloading process.

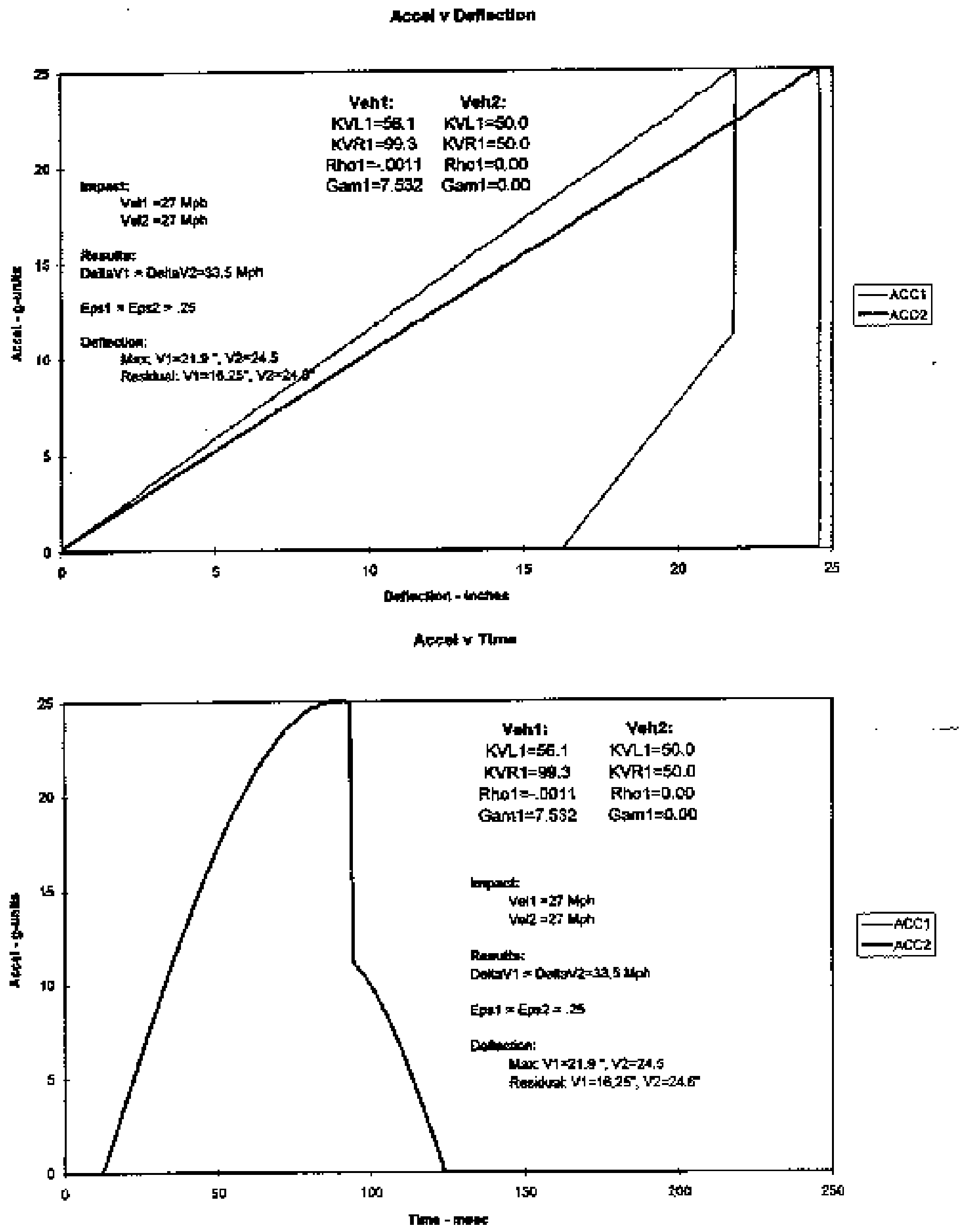


Figure 16 Example Response characteristics of revised SMAC collision routine, 27 MPH Head-On, High and Low Restitution Vehicles.

CONCLUSIONS AND RECOMMENDATIONS

1. Damage analyses which make use of either the original SMAC or CRASH programs produce lower-than-actual values for the impact speed-change, with the maximum errors occurring at low speeds where restitution is greatest. The original CRASH (EDCRASH) does not, of course, include restitution. The simplistic form of simulation of restitution in the original SMAC (EDSMAC), which restores a portion of the crush energy as needed in a time-history solution form, does so in a manner that retains an excessive amount of predicted residual damage. As a result, the error in a purely damage-based ΔV is somewhat greater for SMAC than CRASH at low speeds and somewhat less at high speeds.

2. The developed implementations for both SMAC and CRASH of a revised damage-based procedure that includes restitution have been shown to be capable of producing significant improvements in the accuracy of reconstruction results. They also can serve to insure compatibility of the inputs that define crush properties for the SMAC and the CRASH forms of analysis.

3. The revised damage analysis procedures for CRASH and SMAC provide a unique capability for entering separate definitions of the restitution properties of collision partners.

4. The limited comparisons of the mathematical model of crush behavior with test data that have been possible to date indicate a reasonable degree of correlation. The general form of the mathematical model is dictated by the assumption of a linear relationship between the Delta-V preceding restitution (i.e., to the point of a common velocity) and the residual crush. It should be noted that the specific analytical approach that is defined in Appendices 1 and 2 is not inherently limited to the case of a linear force-deflection characteristic. Modification of the modeled crush behavior to include a saturating force could be readily accomplished if corresponding test data were to become available.

5. The reported research results are considered to constitute an important demonstration of a means of achieving significant improvements in reconstruction accuracy. The restitution aspects of test data needed to fully utilize the described refinements are sometimes included in test reports. They should be made to be a routine part of crash test reports.

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CONTACT INFORMATION

Questions or comments on the paper are welcomed and can be addressed to the authors:

e-mail: **mchenry@interpath.com**

Postal Service Mail: **103 Brady Court, Suite 200**

Cary, NC 27511 USA

WWW: **http://www.mchenrysoftware.com**

APPENDIX 1: Fitting of Crush Properties to Crash Test Data

Definition of Symbols:

CRASH3 Crush Coefficients, Ref. [5]:

A = Intercept, Lb./Inch

B = Slope, Lb./In^2

Campbell Coefficients, Ref. [6]:

b_o = Intercept, Miles per hour

b_1 = Slope, Miles per hour/inch

E_a = Absorbed energy, inch-lb.

E_r = Restored energy, inch-lb.

Figure A1:

F = Crush resistance force, Lb.

F' = Virtual crush resistance force, Lb.

K_1 = Crush resistance per unit width for increasing loads, lb./in^2

K_2 = Crush resistance per unit width for decreasing loads, lb./in^2

K_v = SMAC crush resistance, Lb./in^2 Ref. [**3**]

L = Contact width, inches

M = Mass, lb.-sec^2/in

V_c = Common velocity of contact regions at end of approach period of collision, inches/sec

V_o = Initial velocity, inches/sec.

V_f = Final (separation) velocity, inches/sec

ΔV_c = Impact speed-change during the approach period of the collision, inches/sec

ΔV = Total impact speed-change, inches/sec.

δ = Crush, inches.

δ_f = Residual crush, inches.

δ_m = Maximum dynamic crush, inches

Γ = Restitution constant.

ε = Coefficient of restitution

ρ = Restitution constant.

To the extent that the approach-period impact speed change, ΔV_c, can be reasonably approximated as a linear function of the residual crush, δ_f, the corresponding relationship between the coefficient of restitution, ε, and the maximum dynamic crush, δ_m, must be defined in the form:

$$\varepsilon = \frac{\Gamma}{\delta_m} + \rho \qquad (1)$$

{ if $\varepsilon > 1.00$, $\varepsilon = 1.00$ }

where ρ and Γ are fitted constants.

While the fitted A and B coefficients of CRASH are retained to define ΔV_c as a linear function of δ_f, those coefficients are supplemented by measures from crash tests of the restitution behavior. The combined crash test information is applied in the form of a total of four fitted constants:

K_1, K_2, ρ, Γ

The fitting procedure consists of following:

From measured data in crash tests, a test condition is selected at which the restitution behavior is to be matched. For that selected condition the maximum dynamic crush, $(\delta_m)_1$, the ratio of residual crush to maximum dynamic crush, $(\delta_f/\delta_m)_1$, and the corresponding coefficient of restitution, $(\varepsilon)_1$, are determined to supplement A and B. The four fitted constants are then calculated:

$$K_2/K_1 = \left\{\frac{(\varepsilon)_1}{1-\left(\delta_f/\delta_m\right)_1}\right\}^2 \qquad (2)$$

$$\Gamma = \frac{A}{B}\sqrt{\frac{K_2}{K_1}} \qquad (3)$$

$$\rho = (\varepsilon)_1 - \frac{\Gamma}{(\delta_m)_1} \qquad (4)$$

$$K_1 = B\left(1-\rho\sqrt{\frac{K_1}{K_2}}\right)^2 \ \mathrm{LB/IN^2} \qquad (5)$$

$$K_2 = K_1\left(\frac{K_2}{K_1}\right) \ \mathrm{LB/IN^2} \qquad (6)$$

The lack of exact repeatability of crash test results makes it clearly necessary to rely on multiple crash tests for a definition of crash behavior and to deal with the scatter of the results by means of statistical procedures. Measurements related to restitution behavior have been found to include an unusually large amount of scatter, particularly at low levels of returned energy (for example, see comparisons with test data elsewhere in this paper).

In view of the preceding, it is highly desirable to use regression analysis on multiple data points, to insure that the selected test condition is representative of the overall measurements of behavior. In the case of multiple data points, linear regression is first used to define the coefficients of equation (18) of Appendix 2, and, thereby, A and B. Next, the coefficient of δ_m in equation (9) of Appendix 1 is established by means of a fit to the reported values of δ_f and δ_m with an intercept at -A/B. A value for $(\delta_m)_1$ is selected and the corresponding values of $(\delta_f)_1$ and $(\delta_f/\delta_m)_1$ are calculated from the fitted equation (9). The measured value of $(\varepsilon)_1$, corresponding to $(\delta_m)_1$, is established and entered.

As more measurements of restitution behavior become available, it may become possible to establish patterns in the fitted constants for different vehicle types.

The physical significance of the fitted constants may be seen in Figures A1 and A2.

In Figure A1, it should be noted that the peak values of forces, F and F' are not identical. The plot of F' against deflection must be recognized to constitute a virtual force deflection, since the force and the deflection do not exist at the same time. It can be shown analytically that the ratio of the peak forces in Figure A1 is defined by the following:

$$\left(F/F'\right)_{PEAK\ \mathrm{VALUES}} = \sqrt{\frac{K_1}{B}} \qquad (7)$$

The unit-width crush stiffness for increasing loads can also be established from the following relationship, which is independent of the extent of restored energy and, thereby, of the restitution coefficient:

$$K_1 = \frac{1}{B}\left\{\frac{A}{\left(\delta_m\right)_1} + B\left(\delta_f/\delta_m\right)_1\right\}^2 \quad \mathrm{LB/IN^2} \qquad (8)$$

Sample applications of equation (8) are presented in Fig. A3.

Further analytical relationships include the following:

$$d_f = d_m\sqrt{\frac{K_1}{B} - \frac{A}{B}} \quad \mathrm{INCHES} \qquad (9)$$

$$\delta_f/\delta_m = \frac{\sqrt{K_1/B}}{\left(1 + \frac{A}{B\ \delta_f}\right)} \qquad (10)$$

$$\rho = \sqrt{\frac{K_2}{K_1}}\left(1 - \sqrt{\frac{K_1}{B}}\right) \qquad (11)$$

$$\varepsilon = \frac{\frac{A}{B}\sqrt{\frac{K_2}{B}}}{\left(\delta_f + \frac{A}{B}\right)} + \rho \qquad (12)$$

The relationship of the four fitted constants to A and B may be further defined:

$$B = \frac{K_1}{\left(1 - r\sqrt{\frac{K_1}{K_2}}\right)^2} \quad \mathrm{LB/IN^2} \qquad (13)$$

$$A = B\,\Gamma\sqrt{\frac{K_1}{K_2}} \quad \mathrm{LB/IN} \qquad (14)$$

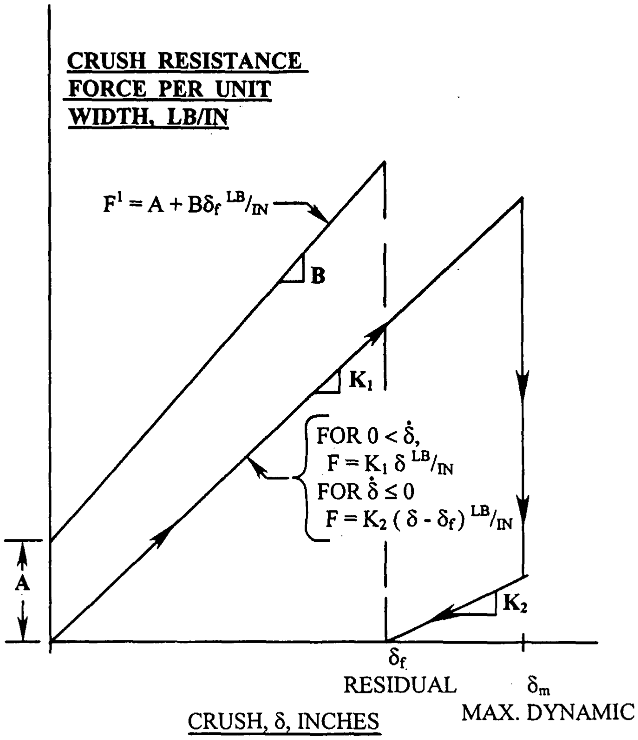


Figure A1: Crush Resistance Force Per Unit Width, Lb/In

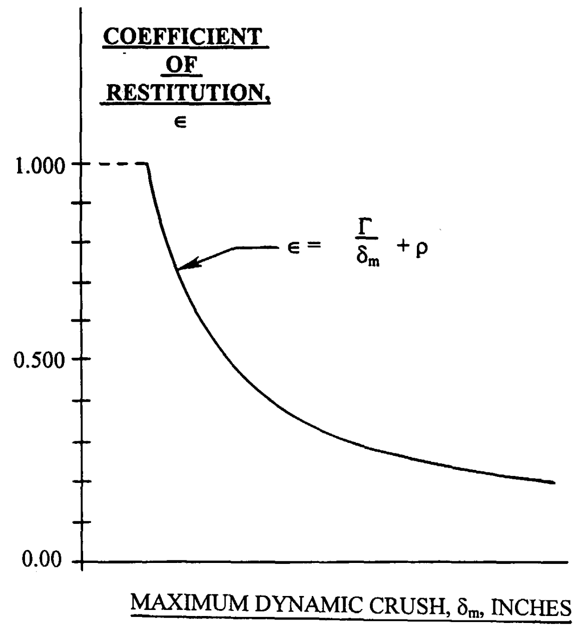
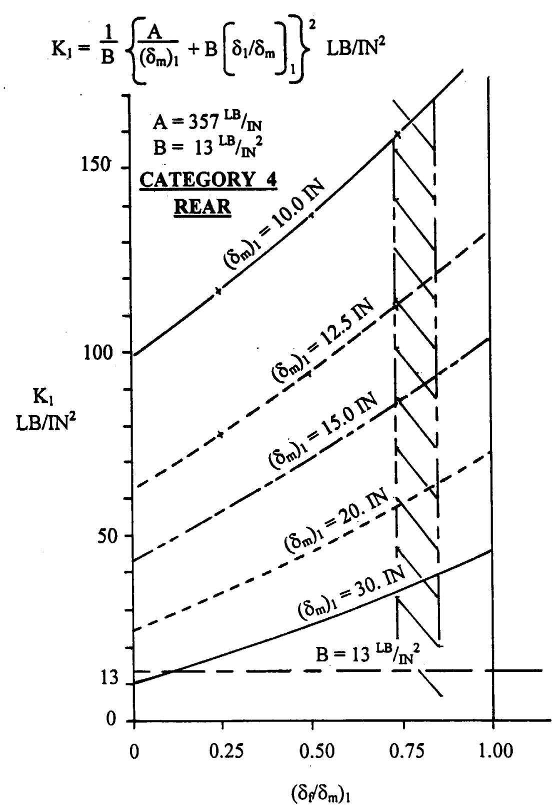


Figure A2: Coefficient of Restitution, e



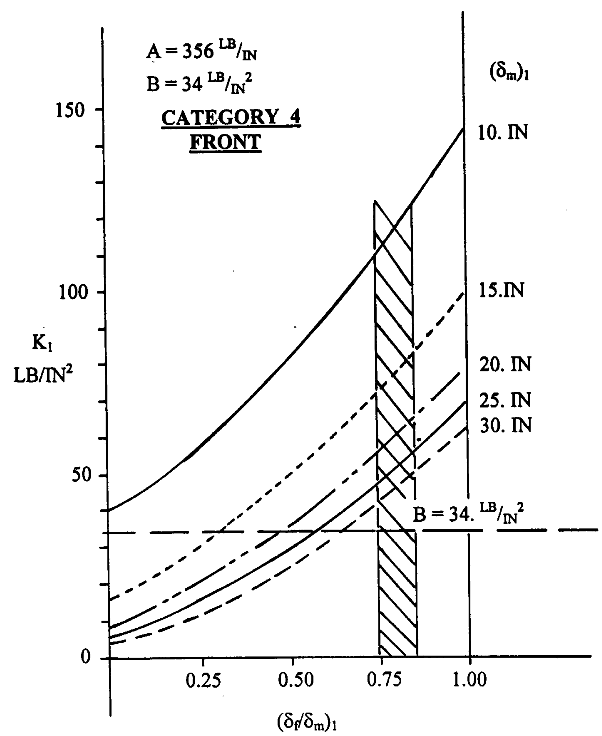


Figure A3: Loading Stiffness vs. Residual/Max Crush Ratio

APPENDIX 2: Derivation of Equations

From Figure A1, the ratio of returned to absorbed energy may be expressed:

$$\frac{E_R}{E_A} = \frac{K_2\left(\delta_m - \delta_f\right)^2}{K_1 \delta_m^2} = \frac{K_2}{K_1}\left(1 - \frac{\delta_f}{\delta_m}\right)^2 \quad (15)$$

Since the coefficient of restitution is equal to the square root of the energy ratio,

$$\varepsilon = \sqrt{\frac{E_R}{E_A}} = \left(1 - \frac{\delta_f}{\delta_m}\right)\sqrt{\frac{K_2}{K_1}} \quad (16)$$

Solving (16) for δ_f,

$$\delta_f = \delta_m\left(1 - \varepsilon\sqrt{\frac{K_1}{K_2}}\right) \quad (17)$$

In order for ΔV_c at a given value of residual crush, δ_f, to be equal in the CRASH and SMAC forms of analysis (see Figure A1), the relationship between δ_f and δ_m must be defined by the following two equations:

$$\Delta V_c = A\sqrt{\frac{L}{BM}} + \delta_f\sqrt{\frac{BL}{M}} \quad (18)$$

$$\Delta V_c = \delta_m\sqrt{\frac{K_1 L}{M}} \quad (19)$$

From (18) and (19),

$$\delta_f = \delta_m\sqrt{\frac{K_1}{B}} - \frac{A}{B} \quad (20)$$

$$\frac{d\delta_f}{d\delta_m} = \sqrt{\frac{K_1}{B}} \quad (21)$$

From equation (17),

$$\frac{d\delta_f}{d\delta_m} = \frac{\delta_f}{\delta_m} - \delta_m\sqrt{\frac{K_1}{K_2}}\frac{d\varepsilon}{d\delta_m} \quad (22)$$

$$\frac{d^2\delta_f}{d\delta_m^2} = -\sqrt{\frac{K_1}{K_2}}\left(\delta_m\frac{d^2\varepsilon}{d\delta_m^2} + 2\frac{d\varepsilon}{d\delta_m}\right) \quad (23)$$

Since $\frac{d\delta_f}{d\delta_m} = \text{constant}$, $\frac{d^2\delta_f}{d\delta_m^2} = 0.$

Solution of (23), for $\frac{d^2\delta_f}{d\delta_m^2} = 0$, yields:

$$\varepsilon = \frac{\Gamma}{\delta_m} + \rho \quad (24)$$

From (24), (22), and (21),

$$\frac{\delta_f}{\delta_m} + \frac{\Gamma}{\delta_m}\sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{K_1}{B}} \quad (25)$$

From (20)

$$\frac{\delta_f}{\delta_m} = \sqrt{\frac{K_1}{B}} - \frac{A}{B\delta_m} \quad (26)$$

Substitution of (26) into (25) yields:

$$\Gamma = \frac{A}{B}\sqrt{\frac{K_2}{K_1}} \quad (27)$$

The ratio K_2/K_1 is defined on the basis of equation (16):

$$\frac{K_2}{K_1} = \left(\frac{\varepsilon_1}{1 - \left(\delta_f/\delta_m\right)_1}\right)^2 \quad (28)$$

where $(\varepsilon)_1$ and $(\delta_f/\delta_m)_1$ are obtained from crash test data for the specific vehicle for which crush properties are being defined.

From equation (24),

$$\rho = (\varepsilon)_1 - \frac{\Gamma}{\left(\delta_m\right)_1} \quad (29)$$

where $(\varepsilon)_1$ and $(\delta_m)_1$ are obtained from crash test data for the specific vehicle for which crush properties are being defined.

From equations (25), (17) and (24):

$$K_1 = B\left\{1 - r\sqrt{\frac{K_1}{K_2}}\right\}^2 \quad \mathbf{(30)}$$

APPENDIX 3: Clarifications

The review process has revealed several points that may be confusing to some readers. The following is aimed at clarification of those aspects of the paper:

1. The existence of <u>significant</u> <u>errors</u> related to restitution that are present in both the original CRASH (EDCRASH) and original SMAC (EDSMAC) computer programs must be acknowledged. It is common knowledge that the original CRASH (EDCRASH) program underestimates the ΔV in barrier crashes by approximately 10 to 20% at 30 MPH and by a greater amount at lower speeds as a result of the fact that restitution is completely ignored. The original SMAC (EDSMAC) form of simulation of restitution is crude, with rarely changed inputs, and it cannot rationally be expected to produce reliable and accurate combinations of dimensional recovery and partial return of absorbed energy for all vehicles under all collision conditions. In fact, its under-prediction of structural recovery produces a similar range of underestimates of ΔV. Thus, the status quo regarding restitution effects in existing computer programs is very difficult to defend on a logical basis.

2. The ranges of errors (underestimates) indicated in the paper for damage-based ΔV values from the original CRASH and SMAC programs, that are produced either by a total neglect of restitution (CRASH) or by under-prediction of structural recovery (SMAC) are intended to provide the reader with approximate measures of the practical significance of the effects of restitution.

3. On the basis of Figures 2, 6A, and A2 as well as the closely related SAE 861894 it should be clear that ε is set to 1.000 whenever the calculated value exceeds 1.000. Therefore any concern about an "infinite limit" for ε is unfounded.

4. The modeled value of ε, at zero residual crush, is not necessarily equal to 1.000. Rather, it is defined by equation (16):

$$\varepsilon = \sqrt{\frac{E_R}{E_A}} = \left(1 - \frac{\delta_f}{\delta_m}\right)\sqrt{\frac{K_2}{K_1}}$$

Results of applications of equation (16) to test data where the limiting value of ε was substantially less than 1.00 are shown in Figures 13 and 14. In Figure 13, the restitution coefficient for the Escort never exceeds 0.314. In calculations related to Figure 13 performed by one reviewer the value of ε was incorrectly set equal to 1.00.

5. Energy is absorbed whether or not a structure is elastic. It is the extent of return of the absorbed energy that distinguishes elastic from inelastic behavior.

6. The term "damage" in this paper is used to refer to the generally accepted residual crush, as opposed to any cosmetic disfigurement of "fragile body parts".

7. The general form of the A,B crush coefficients used in the CRASH(EDCRASH) program implies an effective elastic deformation range, in terms of full dimensional recovery, equal to A/B.

8. The lack of exact repeatability of individual measurements in crash tests that are performed under identical test conditions acts to produce ranges of measured responses rather than single values. For this reason, rigorous measures of reconstruction accuracy must be based on the mean experimental measurements for a given set of test conditions.

9. Available experimental measurements of restitution behavior over ranges of impact speed (e.g., Figures 12, 13, and 14) include only single measured values at the individual test conditions. Clearly, progress toward a rigorous and complete validation study is data-limited at the present time.