Notes on Interpretation of Collision Damage

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The effect of a non-zero velocity of the striking vehicle subsequent to a collision on the extent of damage to that vehicle for a given speed change continues to be a topic about which erroneous and/or misleading statements are frequently made (e.g., References 1, 2 and 3). The general nature of related discussions is indicated in the following paragraph.

For a collision speed change (ΔV) of the striking vehicle of 50 MPH occurring between 100 MPH and 50 MPH. it is asserted that the structure dissipates three times as much energy as in the case of the same speed change occurring between 50 MPH and 0 MPH. Further, it is argued that a substantially greater structural deformation in the former case can lead to an erroneous overestimate of the severity of the occupant exposure. In Reference 2, an impact with a "solidly constructed heavy lorry" is used as an illustrative example of the "nearest real-life equivalent to a moving crash barrier". For a collision of this type, the conclusions of References 1 and 2 are clearly erroneous. In Reference 3, statements regarding relationships between energy dissipation and ΔV are made which, for collisions in general, are not true. They are followed by an illustrative example, which is the somewhat special case of a sideswipe against a fixed barrier.

If such discussions and the related conclusions were correct for collisions in general, it would be necessary to know not only the mass and stiffness of the struck object but also the speed range in which the damage occurred before damage to the striking vehicle could be interpreted in terms of occupant exposure severity. Since the SMAC (Reference 4) and CRASH (Reference 5) computer programs are based on an assumption that the damage for a given ΔV is independent of the speed range in which the ΔV occurs, it was considered to be essential to attempt to clarify the involved physical principles.

Energy Considerations

Perhaps the simplest approach to the topic is a straightforward analysis of the total change in system energy that occurs as the result of a collision between two bodies. It must be recognized that in order to achieve a 100 MPH to 50 MPH speed change in a non-sideswipe collision, the struck object must be accelerated from its initial speed to the common speed of 50 MPH in the direction of motion of the striking vehicle. Therefore, in Figure 1, the following relationships must exist.

Initial Kinetic Energy of System,

$$(KE)_1 = \frac{1}{2} M_1 V_{10}^2 + \frac{1}{2} M_2 V_{20}^2 . \tag{1}$$

Final Kinetic Energy of System,

$$(KE)_2 = \frac{1}{2} (M_1 + M_2) V_c^2 . \tag{2}$$

From Conservation of Momentum,

$$M_1 V_{10} + M_2 V_{20} = (M_1 + M_2) V_c . \tag{3}$$

Solution of (3) for the required initial speed of the struck vehicle, V_{20}, as a function of the initial velocity of the striking vehicle, V_{10}, and the common velocity, V_c, yields

$$V_{20} = \frac{1}{M_2} [(M_1 + M_2) V_c - M_1 V_{10}] . \tag{4}$$

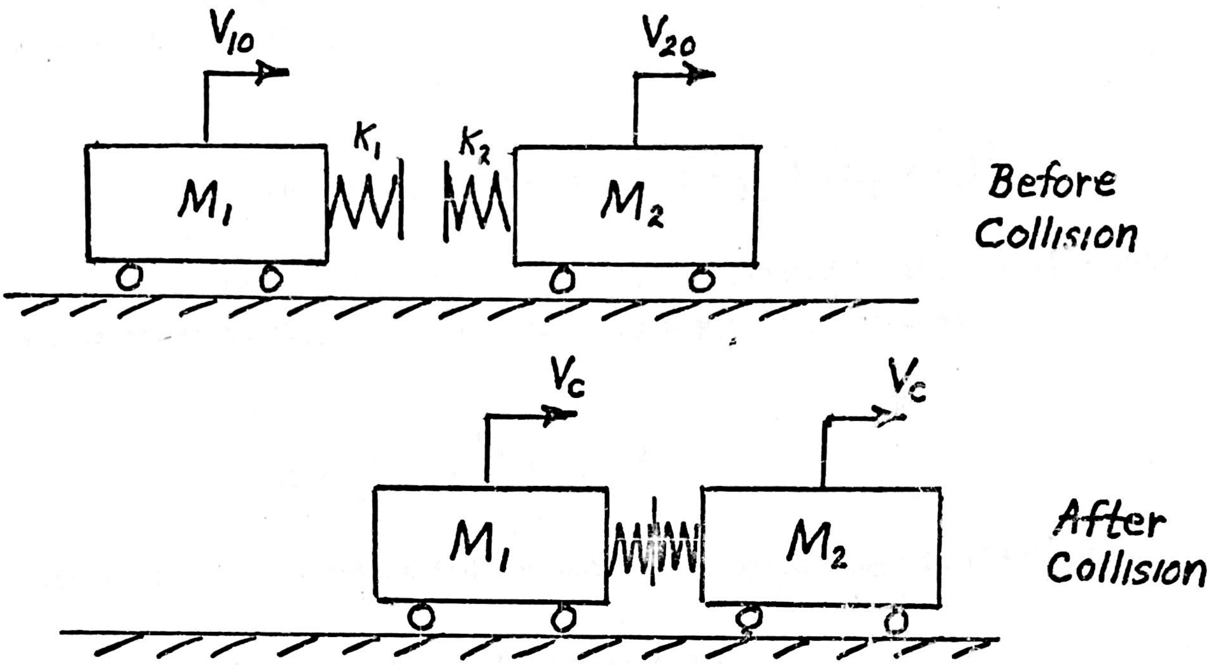

Figure 1 SCHEMATIC SKETCH OF COLLIDING BODIES

Substitution of (4) in (1) and subtraction of (2) from (1) yields the following expression for the total amount of system energy dissipated during the collision,

$$DE = (KE)_1 - (KE)_2 = \frac{M_1}{2} (1 + \frac{M_1}{M_2}) (V_{10} - V_c)^2. \quad (5)$$

Since the speed change, (ΔV_1), experienced by the striking vehicle, M_1, is equal to $(V_{10} - V_c)$, equation (5) may be expressed as,

$$DE = \frac{M_1}{2} (1 + \frac{M_1}{M_2}) (\Delta V_1)^2. \quad (6)$$

Note that equation (6) is independent of the speed range in which ΔV_1 occurs.

In the case of an SAE barrier crash,

$$M_2 \to \infty \text{ and } V_{20} = 0.$$

Therefore, $V_c = 0$, $(KE)_2 = 0$, and

$$(DE)_B = \frac{M_1}{2} (\Delta V_1)^2 . \quad (7)$$

From equations (6) and (7) it may be seen that the total dissipation of system energy in the case of a collision of a given speed change with an obstacle of finite mass (i.e., movable) is related to that for the corresponding case of an SAE barrier crash by the following ratio

$$\frac{DE}{(DE)_B} = 1 + \frac{M_1}{M_2} . \quad (8)$$

Note that equation (8) is independent of the speed range in which the energy dissipation (DE) occurs.

The dissipated system energy will be distributed between the two colliding bodies in inverse proportion to their relative stiffnesses. From equation (8), for the case of identical vehicles, a collision in which M_1 decelerates from 100 MPH to 50 MPH will produce an energy dissipation twice as large as that in an SAE barrier crash at 50 MPH. However, since the dissipation will distribute equally between the structures of identical vehicles, each vehicle will be damaged to the same extent as it would be in a 50 MPH barrier crash. Note that the stated speed-change conditions of this sample case would require that the equal-mass struck vehicle be initially at rest.

A 100 MPH to 50 MPH collision with a heavier vehicle for which $M_1 << M_2$ will produce a smaller total dissipation of system energy than that in the cited equal-mass case (i.e., a larger portion of the initial system energy will become kinetic energy of the struck vehicle) with the limiting value of the total dissipated energy approaching that of a 50 MPH barrier crash (see Figure 2). In this sample case, it would be necessary for the heavier vehicle to be initially moving in the same direction as the striking vehicle (i.e., for a common velocity of 50 MPH to be achieved). Note that the collision force acting on the heavier vehicle does work by virtue of the fact that the struck vehicle is moving, whereas no work is done on a fixed obstacle.

Force Considerations

For a given speed change of the striking vehicle (ΔV_1) to be produced, the time integral of the applied force must equal the corresponding momentum change of that vehicle (i.e., Newton's Second Law).

$$\int_0^t F\,dt = M_1\,\Delta V_1 \tag{9}$$

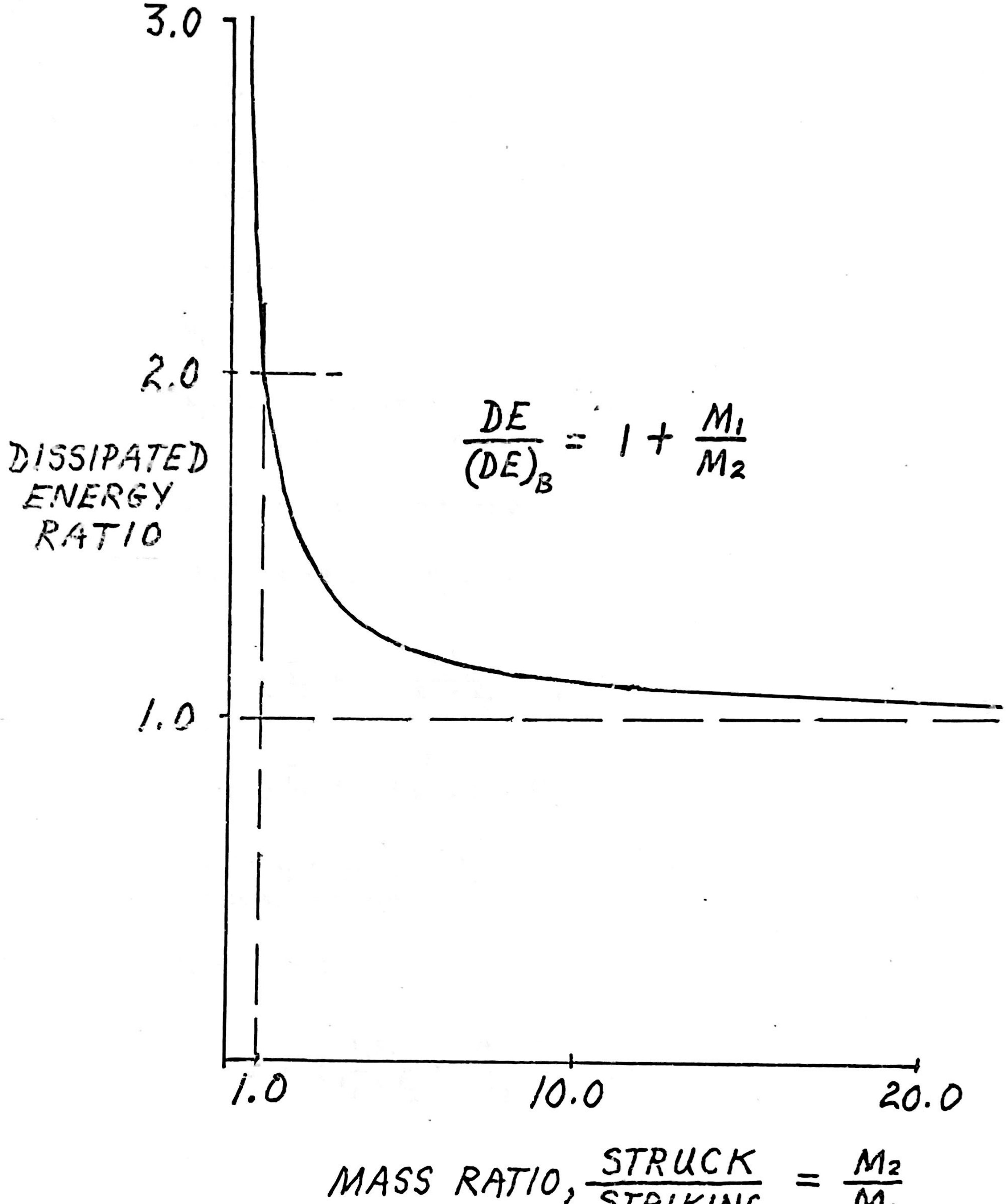

Figure 2 RATIO OF TOTAL DISSIPATED ENERGY FOR $\frac{\text{MOVABLE}}{\text{FIXED}}$ OBSTACLE COLLISIONS, WITH EQUAL SPEED CHANGES OF STRIKING VEHICLE, VS MASS RATIO

If it is assumed for simplicity that the force-deflection characteristics of the two vehicles are each approximately linear for increasing load, the following analysis can provide relationships with which to evaluate the implications of equation (9).

In Figure 3, the symbols are defined as follows.

M_1, M_2 = Masses of Vehicles 1 and 2, lb sec^2/in.

K_1, K_2 = Peripheral crush stiffnesses of Vehicles 1 and 2 for increasing load, lb/in.

X_1, X_2 = Displacements of centers of mass, inches.

The accelerations of M_1 and M_2 can be expressed

$$M_1 \ddot{X}_1 = -\left(\frac{K_1 K_2}{K_1 + K_2}\right)(X_1 - X_2) \tag{10}$$

$$M_2 \ddot{X}_2 = \left(\frac{K_1 K_2}{K_1 + K_2}\right)(X_1 - X_2) \tag{11}$$

Let $\delta = X_1 - X_2$, $\dot{\delta}_o = \dot{X}_{10} - \dot{X}_{20} = V_{10} - V_{20}$, where V_{10}, V_{20} = initial velocities, inches/sec.

From (10) and (11),

$$\ddot{\delta} + \left(\frac{K_1 K_2}{K_1 + K_2}\right)\left(\frac{M_1 + M_2}{M_1 M_2}\right)\delta = 0. \tag{12}$$

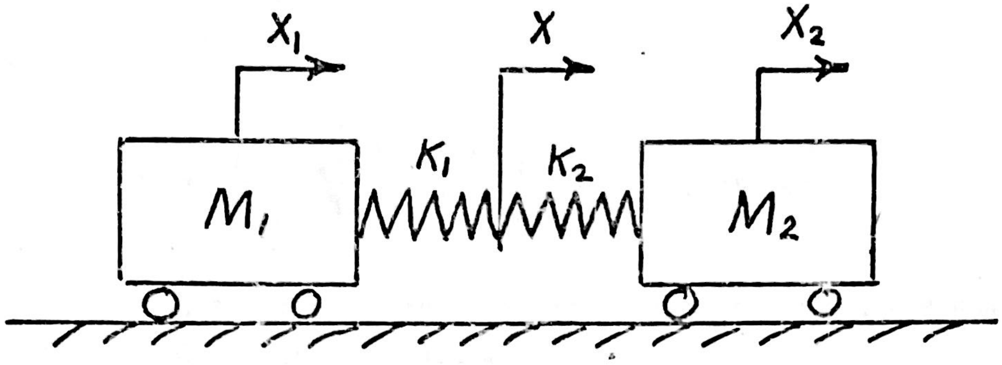

Figure 3 SCHEMATIC SKETCH FOR ANALYSIS OF DYNAMICS

Solution of (12) yields the following relationships:

Response Frequency,

$$f = \frac{1}{2\pi} \sqrt{\frac{K_1 \ (1 + M_1/M_2)}{M_1 \ (1 + K_1/K_2)}} \quad \text{cycles/sec} \tag{13}$$

From (13), the time to reach a common velocity (i.e., one quarter of a complete cycle) can be expressed as

$$t = \frac{1}{4} \left(\frac{1}{f} \right) = \frac{\pi}{2} \sqrt{\frac{M_1 \ (1 + K_1/K_2)}{K_1 \ (1 + M_1/M_2)}} \quad \text{seconds} \tag{14}$$

It may be seen in equation (13) that the response frequency of the two mass system decreases as the obstacle mass (i.e., the struck vehicle mass) increases and/or its stiffness decreases. Therefore, from equation (14), the time to reach a common velocity increases as the obstacle mass increases and/or its stiffness decreases. Note that equation (14) is independent of the speed range in which the collision occurs. The effects of the cited two struck vehicle variables on the time to reach a common velocity are depicted in Figure 4.

From the preceding considerations, it is obvious that the required maximum value of the force, F, in equation (9) to achieve a given speed change of the striking vehicle will decrease as the obstacle mass increases and/or its stiffness decreases, since the effects of such changes act to increase the time interval during which the impulsive force acts. In other words, the production of a given speed change of the striking vehicle by means of impacts with different obstacles requires equal values for the integral, $\int F \, dt$. If the obstacle mass is small, the duration, t, is also small and the force, F, must be larger than that generated by a more massive obstacle.

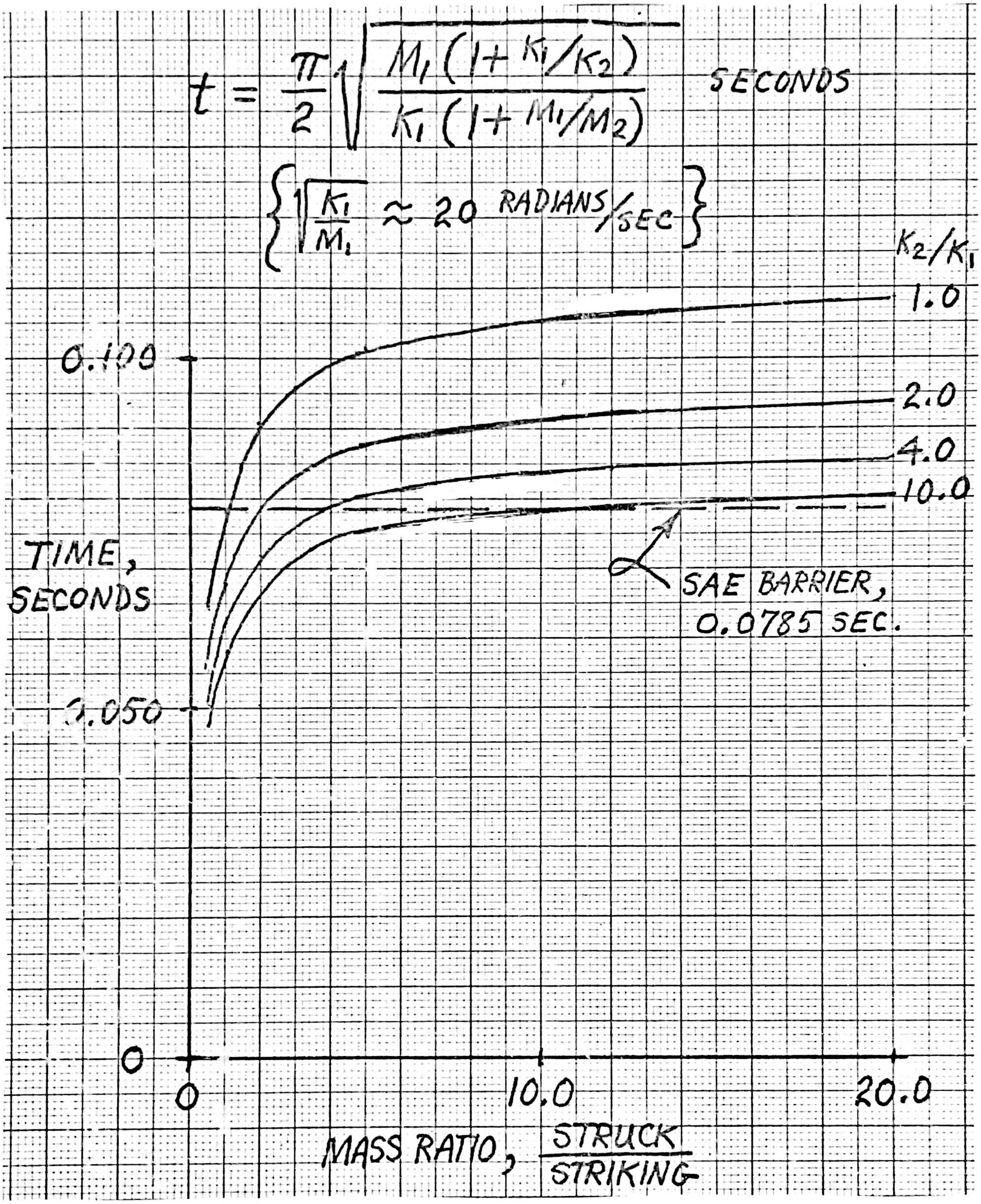

Figure 4 TIME TO REACH A COMMON VELOCITY

The reduction in the required maximum force that is produced by obstacle mass increases and/or stiffness decreases, for a given speed change of the striking vehicle, are also clearly indicated by a solution of (12) for the maximum value of the deflection, δ. (Note that the force, F, is herein assumed to be proportioned to the deflection, for increasing loads.)

$$(\delta)_{max} = (V_{10} - V_{20}) \sqrt{\frac{(K_1 + K_2)\, M_1\, M_2}{K_1\, K_2\, (M_1 + M_2)}} \quad \text{inches} \qquad (15)$$

In Figure 3, let $\delta_1 = X_1 - X$, $\delta_2 = X - X_2$. For force equilibrium, $K_1\, \delta_1 = K_2\, \delta_2$, and by definition $\delta_1 + \delta_2 = \delta$.

Therefore, the deflection, δ_1, of the striking vehicle may be expressed as

$$\delta_1 = \left(\frac{K_2}{K_1 + K_2}\right) \delta \qquad (16)$$

From (15) and (16),

$$(\delta_1)_{max} = (V_{10} - V_{20}) \sqrt{\frac{K_2\, M_1\, M_2}{K_1 (K_1 + K_2)(M_1 + M_2)}} \quad \text{inches} \qquad (17)$$

From Conservation of Momentum, the common velocity, V_c, may be obtained.

$$V_c = \frac{M_1 V_{10} + M_2\, V_{20}}{M_1 + M_2} = \frac{V_{20} + \frac{M_1}{M_2} V_{10}}{1 + M_1/M_2} \qquad (18)$$

Equation (18) can be solved for the speed change of M_1.

$$V_{10} - V_c = \frac{(V_{10} - V_{20})}{(1 + M_1/M_2)} \tag{19}$$

Substitution of (19) into (17) yields

$$(\delta_1)_{max} = (V_{10} - V_c) \sqrt{\frac{M_1 \; (1 + M_1/M_2)}{K_1 \; (1 + K_1/K_2)}} \tag{20}$$

Since the speed change, ΔV_1, experienced by the striking vehicle is equal to $(V_{10} - V_c)$, equation (20) may be expressed as

$$(\delta_1)_{max} = (\Delta V_1) \sqrt{\frac{M_1 \; (1 + M_1/M_2)}{K_1 \; (1 + K_1/K_2)}} \quad \text{inches} \tag{21}$$

In the case of an SAE barrier, for which $M_2 \to \infty$, $K_2 \to \infty$, equation (2) becomes

$$(\delta_1)_B = (\Delta V_1) \sqrt{\frac{M_1}{K_1}} \quad \text{inches} \tag{22}$$

The ratio of deformations for movable/fixed obstacle collisions with a given speed change is obtained from equations (21) and (22).

$$\frac{(\delta_1)_{max}}{(\delta_1)_B} = \sqrt{\frac{1 + M_1/M_2}{1 + K_1/K_2}} \tag{23}$$

Plots of results obtained with equation (23) are displayed in Figure 5. Note that the plotted relationship is independent of the speed range in which the collision occurs.

Concluding Remarks

The relationships that have been derived and the associated discussion are believed to clearly establish the fact that the structural damage produced by a given speed change of a vehicle in a non-sideswipe collision is independent of the speed range in which it occurs. Therefore, interpretations of damage in terms of the severity of occupant exposure can be made without knowledge of the final speed of the striking vehicle. The extent of damage does, of course, depend on the mass and stiffness of the struck obstacle. Also, in a more complete and realistic analysis, the effects of non-central impact configurations must be considered (e.g., see Reference 5).

It appears that the existing confusion related to this topic stems from a lack of recognition of the fact that work is done in accelerating a movable struck obstacle to the common velocity. As a result of this oversight, the total kinetic energy loss of the striking vehicle is erroneously expected to appear in the form of dissipation through structural damage. In view of the use of a collision with a moving truck as an illustrative example in Reference 2, it is speculated that effects of underride may in some cases have been misinterpreted as a confirmation of the expected damage increase. For the special case of a sideswipe, the interpretation of damage must include consideration of the fact that no common velocity was reached.

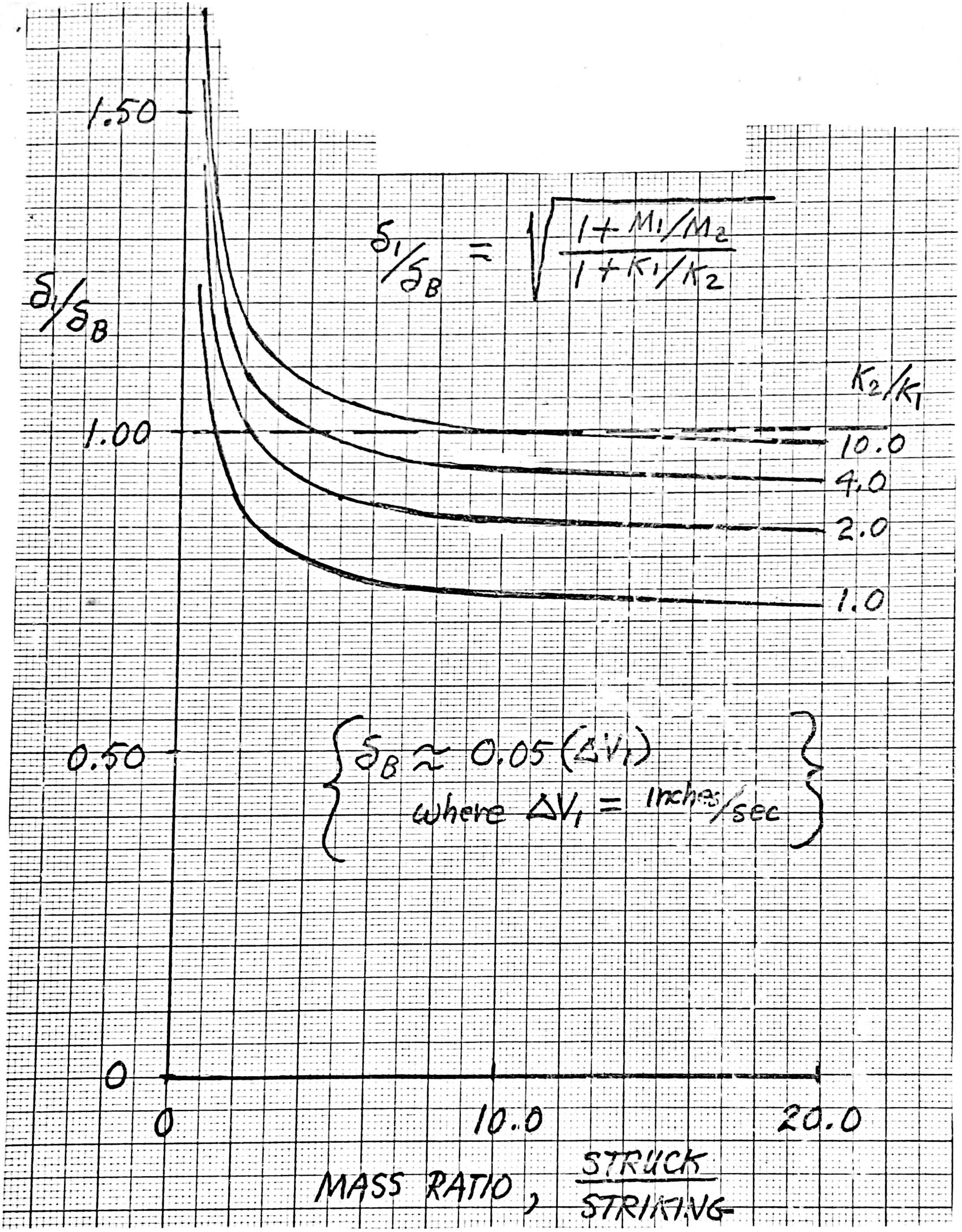

Figure 5 STRIKING VEHICLE - RATIO OF DEFORMATIONS FOR $\frac{\text{MOVABLE}}{\text{FIXED}}$ OBSTACLE, ΔV = CONSTANT

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